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Social Influences and Dropout Risks Related to College Students' Academic Performance: Mathematical Insights

Liqiong Ma[1], W.O. Apeanti[1], H. Prince[1], Hong Zhang[2] *, Houqing Fang[1]

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[1] Faculty of Science, Jiangsu University, ZhenJiang, Jiangsu 212013, PR China
[2] School of Economics and Management, Changzhou Institute of Technology, Changzhou, Jiangsu 213032, PR China

Abstract: How students cope with course failure affects their persistence and dropout from undergraduate programs. To investigate the role of negative social influences on the dynamics of student performance together with dropout from undergraduate studies, we formulate a mathematical model which involves a system of non-linear ordinary differential equations. The existence and stability of both the makeup-free equilibrium (ideal equilibrium) and resit-free equilibrium (subideal equilibrium) are established in terms of threshold parameters, R_{01} and R_{02}, respectively, closely related to the basic reproduction number of mathematical epidemiology. The model is shown to have a backward bifurcation at $R_{02} = 1$ if certain conditions are satisfied, amounting to low values of dropout rates and of negative social influences. This result is also supported through numerical evidence. To establish guidelines for effective policies to eliminate dropout, sensitivity analyses are performed for both R_{01} and R_{02}, the findings being then used to suggest concrete educational measures.

Keywords: Makeup; Resit; Dropout; Social influence; Epidemic model

1 Introduction

The academic performance of undergraduate students has a significant impact upon their dropout intentions. There is ample evidence suggesting that those intentions are prompted, among other factors, by low academic performance and course failure [1]. This happens since, especially for undergraduate students, failing a course represents an acutely negative event, being consequently a major contributor to dropout [2]. To complete their undergraduate education, students must be able to successively meet all minimum requirements for academic progression and also all subsequent requirements for final graduation. To do that, they must fulfill all their course work and obtain the minimum grade point average (GPA) required by their institutions. In the United States for instance, the minimum GPA required for progression and graduation in undergraduate programs is 2.0, a score which implies that the student must get an average grade of C [3]. Students who fail to maintain the minimum GPA for two semesters are usually required to formally withdraw from the university [4, 5].

According to Bandura's social cognitive theory of psychological functioning [6, 7], most human learning occurs in social environments. This means that people acquire skills, strategies, beliefs, norms, and attitudes by interacting with others. Accordingly, the interaction of students with their peers influence their decisions in regard to persistence and dropout [8]. A well known theory of dropout in higher education by Tinto [9, 10] indicates that social integration, through formal extracurricular activities or informal peer group interactions, can influence a student's decision to continue studying or to dropout.

Reducing poor academic performance and dropout risk of undergraduate students is a pressing issue in higher education [11]. Some research studies on student persistence and dropout in higher education have focused on identifying institutional and individual characteristics which are related to students' decision to dropout [12, 13]. Studies have also been conducted on early detection and prediction of students' dropout using statistical and data mining techniques [14]. However, there are still several aspects in this field of research that warrant further exploration. Only a few studies have

* **Corresponding author**. *E-mail address*: cnczzhanghong@163.com

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attempted to investigate how students cope with examination failure, which is a prime cause of their decision to dropout. In this work, using a mathematical model based on a system of nonlinear ordinary differential equations, we shall analyze the negative social influences on the dynamics of student performance together with dropout from the university. The main goals of this study are as follows.

- To propose and analyze a mathematical model for the dynamics of undergraduate student performance that identifies the social attributes related to student dropout risk.

- To determine an accurate threshold value that, once reached, will gradually ease the problem of more and more students fail the makeup examinations.

The remaining part of this paper is organized as follows. In Section 2, we formulate the mathematical model and then establish its basic well-posedness property. In Section 3, the existence of the equilibria is also investigated in terms of threshold parameters, introduced *ad hoc* and closely related to the basic reproduction number often used in mathematical epidemiology. Also the stability analysis are finished. In Section 4, the existence of the backward bifurcation is proved. Parameter estimations are given in Section 5. The sensitivity of the threshold parameters with respect to the parameter values is discussed in Section 6. In Section 7, we numerically illustrate the applicability of the model to a real-life situation. Finally, a discussion of previous results and several concluding remarks are given in Section 8.

2 Model formulation

In all universities, academic regulations require that students who fail a course have to take a make-up examination. If they pass the makeup examination, then they can get the credits of this course. However, if students fail the makeup examination, they must resit the failed course, which means that students have to re-register for that course, attend lectures and re-take the examination. Furthermore, if students fail a number of resit courses, then they are requested to withdraw from the university. Also, students who are dropped out of the university as a result of poor academic performance should leave campus to find a job or pursue their higher education elsewhere.

To determine the impact of social influences between individuals in pass, make-up and resit groups, we shall formulate a model based on the standard SIR epidemic model, in which the undergraduate students, whose size is $N(t)$ at the time t, are divided into three disjoint groups: students who pass all courses $P(t)$, students who have to take make-up examinations $M(t)$, students who have to resit courses $R(t)$.

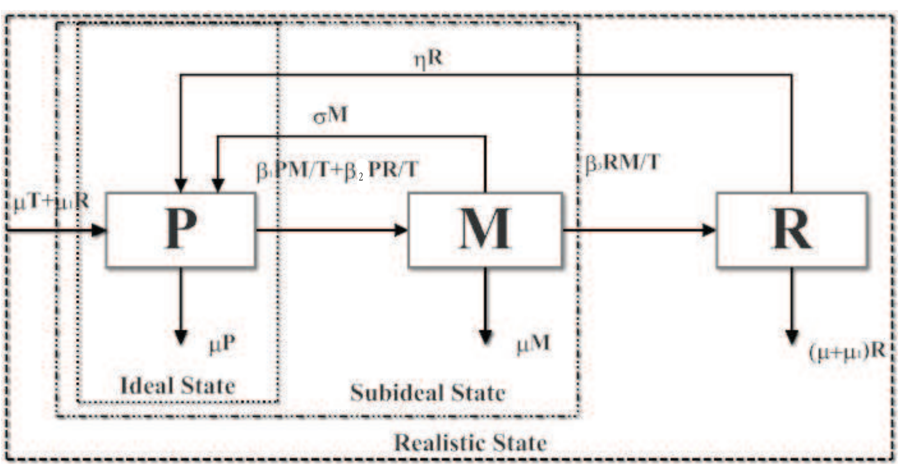

Figure 1: A schematic diagram for our model.

As shown in Fig. 1, the pass population receives incoming students at a rate $\mu T + \mu_1 R$. Here, $\frac{1}{\mu}$ is the average number of years a typical undergraduate student stays in school. The rate of social influence between pass individuals and make-up or resit individuals is given by $\frac{(\beta_1 M + \beta_2 R)P}{T}$, and the rate of social influence between makeup and resit populations is $\frac{\beta_3 MR}{T}$. Make-up students who pass all their make-up examinations join the pass population at a rate σ, owing to students' high internal locus of control. Resit students who pass their resit courses join the pass population at a rate of η, owing to their self-efficacy. Resit students are dropped out of the university at a rate μ_1. Ideally, in an academic environment, one expects all students are only in the pass populations as the *ideal state*. Furthermore, the *subideal state* refers to the situation in which no student needs resitting any course. However, while in real life certain students pass all their examinations, others have to take make-up examinations, resit courses or dropout. Therefore, we refer to the state in which there are students in all subpopulations of the model as the *realistic state*.

On the basis of the above assumptions, we may construct the following model

$$\begin{aligned} \frac{dP}{dt} &= \mu T + (\mu_1 + \eta)R + \sigma M - \mu P - \frac{\beta_1 PM}{T} - \frac{\beta_2 PR}{T}, \\ \frac{dM}{dt} &= \frac{\beta_1 PM}{T} + \frac{\beta_2 PR}{T} - \frac{\beta_3 MR}{T} - (\sigma + \mu)M, \\ \frac{dR}{dt} &= \frac{\beta_3 MR}{T} - (\eta + \mu + \mu_1)R, \end{aligned} \tag{1}$$

with initial conditions $P(0) = P_0 > 0$, $M(0) = M_0 > 0$, and $R(0) = R_0 \geq 0$. We assume that all model parameters, summarized in Table 1 below, are positive. The positivity of the solutions of the system (1) can easily be established if $P_0 > 0, M_0 \geq 0, R_0 \geq 0$. For the sake of simplicity, we may nondimensionalize the state variables and then rewrite

Table 1: Parameters on the dynamics of undergraduate students performance model (1).

parameter	description
$\frac{1}{\mu}$	average number of years a typical undergraduate student stays in school
μ_1	dropout rate
σ	rate of passing makeup examination owing to level of internal control
η	rate of passing resit course owing to level of self-efficacy
β_1	average effective negative influence of makeup students on pass students (AENIMP factor)
β_2	average effective negative influence of dropout students on pass students (AENIDP factor)
β_3	average effective negative influence of resit students on makeup students (AENIRM factor)

the system into proportions of the total population, denoted with the corresponding small letters. We thereby consider the following system

$$\begin{aligned} \frac{dp}{dt} &= \mu + (\mu_1 + \eta)r + \sigma m - \mu p - \beta_1 pm - \beta_2 pr, \\ \frac{dm}{dt} &= \beta_1 pm + \beta_2 pr - \beta_3 mr - (\sigma + \mu)m, \\ \frac{dr}{dt} &= \beta_3 mr - (\eta + \mu + \mu_1)r, \end{aligned} \tag{2}$$

where p=$\frac{P}{T}$, m=$\frac{M}{T}$, r=$\frac{R}{T}$.

3 Stability analysis

The model is well posed and has positivity-preserving solutions. Also, the model has a makeup-free equilibrium (understood as the "ideal" equilibrium), given by

$$X_0 = (p_0, m_0, r_0) = (1, 0, 0).$$

We now introduce a quantity which is similar in its scope to the basic reproduction number of mathematical epidemiology described in [16]. This quantity, the makeup reproduction number $\mathcal{R}_{01}$ of the model, is defined herein as the average number of pass students are influenced by one member of makeup population to fail the courses. After some simple calculations we obtain our next generation matrix

$$FV^{-1} = \begin{pmatrix} \frac{\beta_1}{\sigma+\mu} & \frac{\beta_2}{\eta+\mu+\mu_1} \\ 0 & 0 \end{pmatrix}.$$

The basic reproduction number is the maximum eigenvalue of the matrix above and given by

$$R_{01} = \frac{\beta_1}{\sigma + \mu}.$$

The Jacobian matrix $J(X_0)$ evaluated at X_0 is

$$J(X_0) = \begin{pmatrix} -\mu & \sigma - \beta_1 & \mu_1 + \eta - \beta_2 \\ 0 & \beta_1 - \sigma - \mu & \beta_2 \\ 0 & 0 & -\eta - \mu - \mu_1 \end{pmatrix}. \quad (3)$$

The characteristic polynomial for the Jacobian matrix atX_0 is

$$(\lambda + \mu)[\lambda - (\beta_1 - \sigma - \mu)](\lambda + \eta + \mu + \mu_1) = 0.$$

Then the eigenvalues of the Jacobian matrix are

$$\lambda_1 = -\mu < 0, \lambda_2 = \beta_1 - \sigma - \mu, \lambda_3 = -(\eta + \mu + \mu_1) < 0.$$

Hence, we get the following result.

Theorem 1 *The ideal state is locally asymptotically stable if and only if* $R_{01} < 1$.

The model (2) has a resit-free equilibrium (understood as the "subideal" state), given by

$$X_1 = (p_1, m_1, r_1) = (\frac{\sigma + \mu}{\beta_1}, 1 - \frac{\sigma + \mu}{\beta_1}, 0)$$

provided that $R_{01} > 1$.

Similarly, the resit reproductive number is defined as the average number of makeup students are influenced by one member of resit population to resit the courses and given by

$$R_{02} = \frac{(\beta_1 - \sigma - \mu)\beta_3}{(\mu_1 + \mu + \eta)\beta_1} = (\frac{\beta_3}{\mu_1 + \mu + \eta})(1 - \frac{1}{R_{01}}).$$

The Jacobian matrix $J(X_1)$ evaluated at X_1 is

$$J(X_1) = \begin{pmatrix} \sigma - \beta_1 & -\mu & \frac{(\mu_1+\eta)\beta_1 - \beta_2(\sigma+\mu)}{\beta_1} \\ \beta_1 - \sigma - \mu & 0 & \frac{(\beta_2+\beta_3)(\sigma+\mu) - \beta_1\beta_3}{\beta_1} \\ 0 & 0 & \frac{\beta_3(\beta_1-\sigma-\mu) - \beta_1(\eta+\mu+\mu_1)}{\beta_1} \end{pmatrix}. \quad (4)$$

The characteristic polynomial for the Jacobian matrix atX_1 is

$$(\lambda - [\frac{\beta_3(\beta_1 - \sigma - \mu) - (\eta + \mu + \mu_1)}{\beta_1}])(\lambda[\lambda - \beta_1 - \sigma] - \mu[\sigma + \mu - \beta_1]) = 0.$$

Then the eigenvalues of the Jacobian matrix are $\lambda_1 = -\mu < 0$, $\lambda_2 = \sigma + \mu - \beta_1 < 0$, and

$$\lambda_3 = \frac{\beta_3(\beta_1 - \sigma - \mu) - \beta_1(\eta + \mu + \mu_1)}{\beta_1}.$$

Then we easily obtain the following result.

Theorem 2 *The subideal state is locally asymptotically stable if and only if* $0 < R_{02} < 1$.

In what follows, we shall find the condition for the existence and uniqueness of the realistic equilibrium (understood as the "realistic" state) of the system (2) is determined. Let X^*=(p^*, m^*, r^*) be the realistic equilibrium. To find the endemic equilibrium, we equate all equations on the right side of the system (2) to zero and then obtain

$$m^* = \frac{\eta + \mu + \mu_1}{\beta_3}, \quad (5)$$

$$p^* = \frac{(\eta + \mu + \mu_1)(\sigma + \mu + \beta_3 r^*)}{\beta_1(\eta + \mu + \mu_1) + \beta_2\beta_3 r^*}, \tag{6}$$

and

$$A(r^*)^2 + Br^* + C = 0, \tag{7}$$

in which

$$\begin{aligned} A &= \beta_2\beta_3^2\mu > 0, \\ B &= \beta_3\mu[(\mu_1 + \mu + \eta)(\beta_1 + \beta_2 + \beta_3) - \beta_2\beta_3], \\ C &= \mu(\mu_1 + \mu + \eta)[\beta_1(\mu_1 + \mu + \eta) + \beta_3(\sigma + \mu - \beta_1)] \\ &= \mu\beta_1(\mu_1 + \mu + \eta)^2(1 - R_{02}). \end{aligned}$$

Let us denote

$$R_{02}^* = 1 - \frac{[(\eta + \mu + \mu_1)(\beta_1 + \beta_2 + \beta_3) - \beta_2\beta_3]^2}{4\beta_1\beta_2(\eta + \mu + \mu_1)^2}.$$

We then obtain the following existence result.

Theorem 3 *The following statements hold.*

(i) *If $R_{02} > 1$, then (2) has a unique realistic equilibrium $I^* = (p^*, m^*, r^*)$, with*

$$r^* = \frac{-B + \sqrt{B^2 - 4AC}}{2A}.$$

(ii) *If $R_{02} = 1$ and $\frac{\beta_1+\beta_2+\beta_3}{\beta_2\beta_3} < \frac{1}{\mu_1+\mu+\eta}$, then (2) has a unique realistic equilibrium $I^* = (p^*, m^*, r^*)$, with*

$$r^* = \frac{-B}{A}.$$

(iii) *If $R_{02} = R_{02}^*$ and $\frac{\beta_1+\beta_2+\beta_3}{\beta_2\beta_3} < \frac{1}{\mu_1+\mu+\eta}$, then (2) has a unique realistic equilibrium $I^* = (p^*, m^*, r^*)$, with*

$$r^* = \frac{-B}{2A}.$$

(iv) *$R_{02}^* < R_{02} < 1$ and $\frac{\beta_1+\beta_2+\beta_3}{\beta_2\beta_3} < \frac{1}{\mu_1+\mu+\eta}$, then (2) has two realistic equilibria $I_1^* = (p_1^*, m_1^*, r_1^*)$ and $I_1^* = (p_2^*, m_2^*, r_2^*)$, with*

$$r_{1,2}^* = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

(v) *If $R_{02} \le 1$ and $\frac{\beta_1+\beta_2+\beta_3}{\beta_2\beta_3} > \frac{1}{\mu_1+\mu+\eta}$, then (2) has no realistic equilibrium.*

(vi) *If $R_{02} < R_{02}^*$, then (2) has no realistic equilibrium.*

4 Backward bifurcation

We perform a backward bifurcation analysis using Theorem 4.1 of Castillo-Chavez and Song [17], a theoretical result based upon the Center Manifold Theory. To use this result, we make the following notations in system (2): $p = x_1$, $m = x_2$ and $r = x_3$. The system (2) can then be written in vector form as

$$\frac{dX}{dt} = H,$$

where $X = (x_1, x_2, x_3)^T$ and $H = (h_1, h_2, h_3)^T$ are transposed matrices. Specifically, the system (2) becomes

$$\begin{aligned}\frac{dx_1}{dt} &= \mu + (\mu_1 + \eta)x_3 + \sigma x_2 - \mu x_1 - \beta_1 x_1 x_2 - \beta_2 x_1 x_3 := h_1,\\ \frac{dx_2}{dt} &= \beta_1 x_1 x_2 + \beta_2 x_1 x_3 - \beta_3 x_2 x_3 - (\sigma + \mu)x_2 := h_2,\\ \frac{dx_3}{dt} &= \beta_3 x_2 x_3 - (\eta + \mu + \mu_1)x_3 := h_3.\end{aligned}$$

Let β_3^* be the bifurcation parameter, when $R_{02} = 1$, $\beta_3 = \beta_3^*$, it has $\beta_3 = \beta_3^* = \frac{\beta_1(\eta+\mu+\mu_1)}{\beta_1-(\sigma+\mu)}$.

The Jacobian matrix $J(X_1)$ evaluated at X_1 is calculated at β_3^* is given by

$$\begin{pmatrix} \sigma - \beta_1 & -\mu & \frac{\beta_1(\eta+\mu_1)-\beta_2(\sigma+\mu)}{\beta_1} \\ \beta_1 - (\sigma+\mu) & 0 & \frac{(\beta_2+\beta_3^*)(\sigma+\mu)-\beta_1\beta_3^*}{\beta_1} \\ 0 & 0 & 0 \end{pmatrix}.$$

It can be shown that $J(X_1)$ has a right eigenvector associated with the eigenvalue 0 at $\beta_3 = \beta_3^*$, given by $w = (w_1, w_2, w_3)^T$ with

$$\begin{aligned} w_1 &= \frac{\beta_1\beta_3^* - (\sigma+\mu)(\beta_2+\beta_3^*)}{\beta_1(\beta_1 - \sigma - \mu)},\\ w_2 &= \frac{(\beta_1 - \sigma - \mu)[\beta_3^*(\sigma - \beta_1) + \beta_1(\mu_1 + \eta)] + \beta_2\mu(\sigma+\mu)}{\mu\beta_1(\beta_1 - \sigma - \mu)},\\ w_3 &= 1.\end{aligned}$$

Similarly, one of left eigenvectors denoted by $v = (v_1, v_2, v_3)$ is $v_1 = v_2 = 0$, $v_3 = 1$. To apply the bifurcation result presented in [17], let us compute the associated parameters involving second order partial derivatives indicated therein, given by

$$\begin{aligned} a &= \Sigma_{k,i,j=1}^3 v_k w_i w_j \frac{\partial^2 h_k}{\partial x_i \partial x_j}(X_1, \beta_3^*),\\ b &= \Sigma_{k,i=1}^3 v_k w_i \frac{\partial^2 h_k}{\partial x_i \partial \beta_3^*}(X_1, \beta_3^*).\end{aligned}$$

So,

$$a = v_3 w_2 w_3 \frac{\partial^2 h_3}{\partial x_2 \partial x_3} + v_3 w_3 w_2 \frac{\partial^2 h_3}{\partial x_3 \partial x_2} = 2\beta_3 w_2 w_3.$$

Also,

$$b = \Sigma_{j=1}^3 v_3 w_j \frac{\partial^2 h_3}{\partial x_i \partial \beta_3^*} = \frac{\eta + \mu + \mu_1}{\beta_3^*} > 0.$$

By applying Theorem 4.1 of Castillo-Chavez and Song [17], we obtain the following result.

Theorem 4 *The realistic equilibrium X^* is locally asymptotically stable for $R_{02} > 1$ but close to 1 and the system (2) has a backward bifurcation at $R_{02} = 1$ if one of the following equivalent conditions is satisfied.*

(i) *The AENIRP factor is greater than a certain critical value, that is,*

$$\beta_2 > \beta_2^{**} = \frac{\beta_1(\beta_1 + \mu_1 + \eta - \sigma)}{\sigma + \mu}.$$

(ii) *The AENIMP factor belongs to some interval, that is,*

$$\beta_1 \in (0, \frac{(\mu_1 + \eta - \sigma) + \sqrt{(\mu_1 + \eta - \sigma)^2 + 4\beta_2(\sigma + \mu)}}{2}).$$

(ii) *The dropout rate is less than a certain critical value, that is,*

$$\mu_1 < (\sigma + \mu)\frac{\beta_2}{\beta_1} + \sigma - \beta_1 - \eta.$$

5 Parameter estimation

To estimate the parameters and validate our model, we design and carry out a survey in a group of undergraduate students from the University of Education, Winneba, Ghana. The sample used here is chosen specifically for demonstration purposes, so the generality of results is not a concern here. The main purpose is to illustrate the potential applications of our model in warding off the negative social influences and then nurturing student progress.

Estimate of μ:

The majority of the students graduate within 4 years. This means that the average time required for graduation is 4 years, leading to μ = 1/(4*365), which is 0.068% per day.

Estimate for μ_1:

The dropout rate is estimated using 20 years enrollment and graduation statistics from the University of Education, Winneba, April, 2016 [18]. From this data, the average dropout rate is determined as being 22.63% per year, which is 0.062% per day.

Estimates of β_1, β_2 ***and*** β_3:

Because of the difficulties in calculating the average rate of negative influence among pass, make-up, and resit students, we use the average number of makeup and resit friends to compute the average number of negative contacts. Assume that lecture sections for undergraduate students start at 8:00 am and end at 9:00 pm, with a total of three hours break during this period. This means that students spend on average 10 hours per day to study together. The rate of negative contacts between pass and makeup students (between pass and resit students, or between makeup and resit students) is computed as the product of average number of negative contacts and the hours of contact per day.

Estimates of σ ***and*** η:

Assume that each student has a unique opportunity to take a makeup examination and a resit examination as well every academic year. The rates of passing makeup examination and resit examination is computed as the ratio of the number of passing students taking this examination to the total number of students taking this examination.

Table 2: Parameters and their base values.

parameter	value	unit
μ	0.0068	day^{-1}
μ_1	0.0062	day^{-1}
σ	0.051	day^{-1}
η	0.018	day^{-1}
β_1	0.15	day^{-1}
β_2	0.24275	day^{-1}
β_3	0.01, 0.2008, 0.4	day^{-1}

6 Sensitivity analysis for R_{01} and R_{02}

We now investigate how R_{01} and R_{02} respond to changes in the parameters, in order to establish effective policies to reduce and perhaps eliminate dropout. The normalized forward-sensitivity index of a variable Q with respect to a parameter p (or the elasticity of Q with respect to p) is defined as

$$\zeta_p^Q = \frac{p}{Q} \cdot \frac{\partial Q}{\partial p}.$$

This index indicates how sensitive Q is to changes of parameter p. Precisely, a positive or negative index indicates that an increase in the parameter value results in an increase or decrease of Q [19].

6.1 The sensitivity index of R_{01} with respect to β_1, μ, and σ

$$\zeta_{\beta_1}^{R_{01}} = \frac{\beta_1}{R_{01}} \cdot \frac{\partial R_{01}}{\partial \beta_1} = 1,$$
$$\zeta_{\mu}^{R_{01}} = \frac{\mu}{R_{01}} \cdot \frac{\partial R_{01}}{\partial \mu} = -\frac{\mu}{\sigma+\mu},$$
$$\zeta_{\sigma}^{R_{01}} = \frac{\sigma}{R_{01}} \cdot \frac{\partial R_{01}}{\partial \sigma} = -\frac{\sigma}{\sigma+\mu}.$$

The normalized forward-sensitivity with respect to each parameter was evaluated using the estimated values given in Table 2. Fig. 2 indicates the sensitivity of R_{01}(≈2.595) with respect to each of the parameters. From the above sensitivity analysis, one notes that $\zeta_{\beta_1}^{R_{01}} = 1.00$, which means that increasing or (decreasing) the negative influence of makeup students on pass students by 10% will result in a corresponding 10% increase in the number of resit students. The sensitivity index $\zeta_{\sigma}^{R_{01}}$ ($\zeta_{\mu}^{R_{01}}$) is positive, which means that increasing the rate of passing makeup examinations increases the number of pass students.

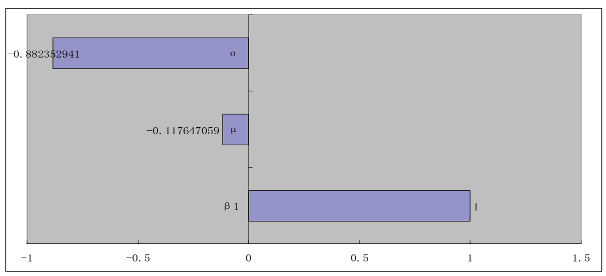

Figure 2: Sensitivity indices for R_{01} against model parameters.

6.2 The sensitivity index of R_{02} with respect to β_1, β_3 μ, μ_1, η and σ

$$\zeta_{\beta_1}^{R_{02}} = \frac{\beta_1}{R_{02}} \cdot \frac{\partial R_{02}}{\partial \beta_1} = \frac{\sigma+\mu}{\beta_1-\sigma-\mu},$$
$$\zeta_{\beta_3}^{R_{02}} = \frac{\beta_3}{R_{02}} \cdot \frac{\partial R_{02}}{\partial \beta_3} = 1,$$
$$\zeta_{\mu}^{R_{02}} = \frac{\mu}{R_{02}} \cdot \frac{\partial R_{02}}{\partial \mu} = \frac{\mu(\sigma-\eta-\mu_1-\beta_1)}{(\beta_1-\sigma-\mu)(\eta+\mu+\mu_1)},$$
$$\zeta_{\mu_1}^{R_{02}} = \frac{\mu_1}{R_{02}} \cdot \frac{\partial R_{02}}{\partial \mu_1} = -\frac{\mu_1}{\eta+\mu+\mu_1},$$
$$\zeta_{\eta}^{R_{02}} = \frac{\eta}{R_{02}} \cdot \frac{\partial R_{02}}{\partial \eta} = -\frac{\eta}{\eta+\mu+\mu_1},$$
$$\zeta_{\sigma}^{R_{02}} = \frac{\sigma}{R_{02}} \cdot \frac{\partial R_{02}}{\partial \sigma} = \frac{\sigma}{\sigma+\mu-\beta_1}.$$

The sensitivity indices of R_{02} (≈1.983) with respect to parameters, evaluated using the estimated values from Table 2, are shown in Fig. 3. From the specific values of the sensitivity indices of R_{02}, one can see that R_{02} is the most sensitive to β_3. Just behind these parameters, R_{02} is also very sensitive to η, σ, and β_1.

7 Numerical simulation

We carried out numerical simulations to investigate the impact of the parameters on the dynamics of the system and on threshold quantities.

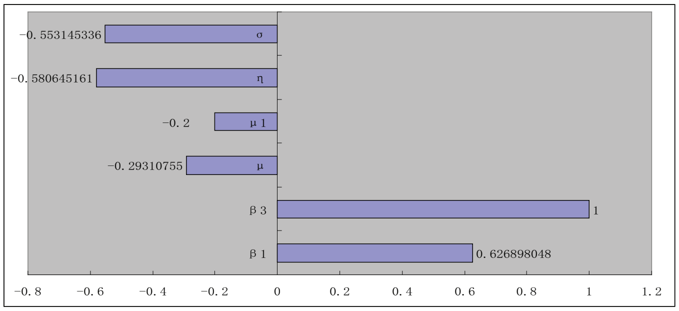

Figure 3: Sensitivity indices for R_{02} against model parameters.

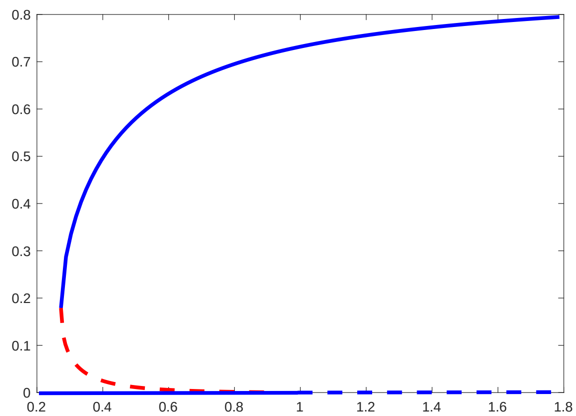

Figure 4: A backward bifurcation. The solid blue curve and line segment denote the stable equilibrium states and the dotted red curve and line segment denote the unstable equilibrium states.

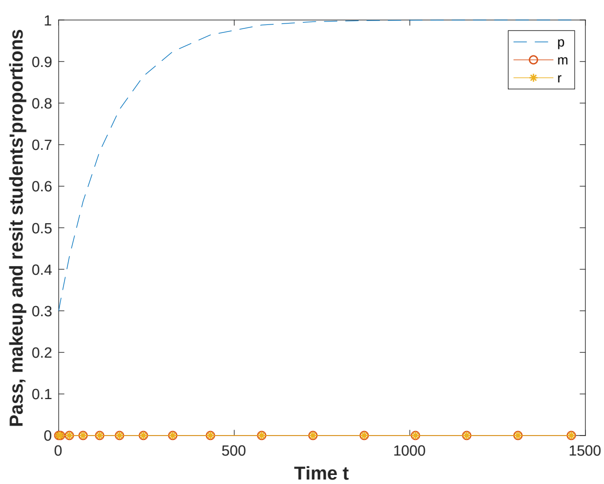

Figure 5: Sensitivity indices for R_{02} against model parameters. Here $\mu = 0.0068$, $\mu_1 = 0.0062$, $\sigma = 0.051$, $\eta = 0.018$, $\beta_1 = 0.035$, $\beta_2 = 0.24275$, $\beta_3 = 0.01$.

To depict a bifurcation curve, we fixed $\mu = 0.0068$, $\mu_1 = 0.0062$, $\sigma = 0.051$, $\eta = 0.018$, $\beta_1 = 0.065$, $\beta_2 = 0.24275$, $\beta_3 = 0.0764$, and then get R^*_{02} approximately equals to 0.2736. As shown in Fig. 4, the backward bifurcation occurs provided that $R_{02} \in (0.2736, 1)$. Our findings suggest that keeping the effective negative influence of resit students on makeup students below a certain threshold may be an efficient way to avoid the backward bifurcation, which would dramatically increase the size of the resit student population. Also, Figs. 5–7 illustrate the respective ideal, subideal and realistic states according to the mathematical results obtained in Section 3.

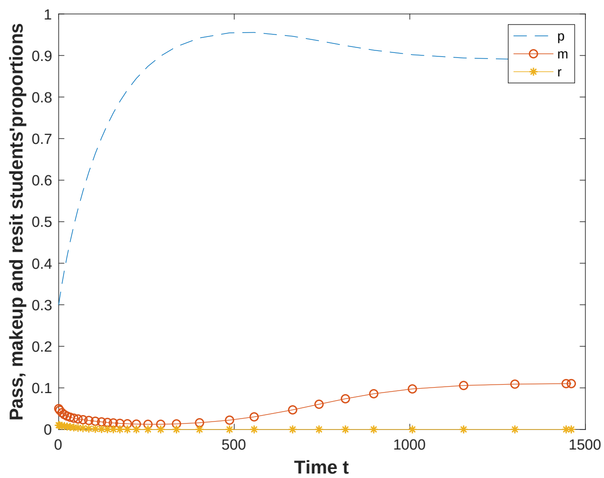

Figure 6: Sensitivity indices for R_{02} against model parameters. Here $\mu = 0.0068$, $\mu_1 = 0.0062$, $\sigma = 0.051$, $\eta = 0.018$, $\beta_1 = 0.035$, $\beta_2 = 0.24275$, $\beta_3 = 0.4$.

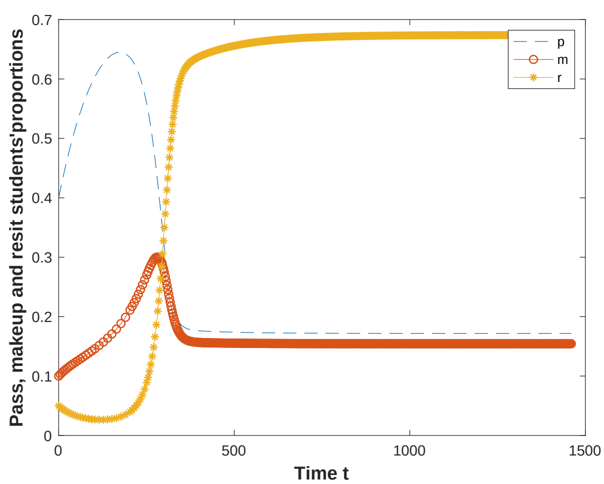

Figure 7: Sensitivity indices for R_{02} against model parameters. Here $\mu = 0.0068$, $\mu_1 = 0.0062$, $\sigma = 0.051$, $\eta = 0.018$, $\beta_1 = 0.035$, $\beta_2 = 0.24275$, $\beta_3 = 0.2008$.

8 Conclusions

In this paper, we developed a bare-bones mathematical model which investigates the effects of negative social influences on students' academic performance together with considering students' dropout from the university. Following the idea of the basic reproduction number of mathematical epidemiology, we thereby define the makeup-free reproduction number

R_{01} as the average number of pass students are influenced by one member of makeup population to fail the courses. Similarly, the resit-free reproductive number R_{02}, which is related to R_{01}, is defined as the average number of makeup students are influenced by one member of resit population to resit the courses. We observed that a backward bifurcation occurs in our model, which shows that decreasing R_{02} below unity will not necessarily decrease the resit student population, even in the long term. In this setting, an effective way of eliminating resit is to adopt new strategies for motivating resit students to study harder and attend lectures. Furthermore, sensitivity analysis have been performed by evaluating the sensitivity indices of the makeup reproduction number (R_{01}) and of the resit reproduction number (R_{02}), respectively, with respect to model parameters. Finally, numerical simulations are given to illustrate our mathematical results. Like any other model, ours is not without limitations either. However, students may makeup, resit or dropout from undergraduate programs for several other unrelated reasons, including the school environment, discipline policies, soci-economic status of students, family dynamics, student learning habits, academic disengagement, behavioural disengagement, student body characteristics, academic policies and demographic characteristics. It is also worth noting that even well-performing students may dropout or change their university for personal reasons. In spite of these limitations, this model presents a unique attempt to model and analyze the dynamics of undergraduate students performance with mathematical tools but from a social perspective associated with considering dropout risks. Future studies can extend this model by incorporating several other factors that could be used to address these limitations.

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