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Secure Communication Based on Synchronization of Three Chaotic Systems

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Abstract: In this paper, we study the synchronization of three chaotic T-system with known and unknown parameters. The nonlinear feedback control and adaptive control schemes are used for synchronization with known and unknown parameters respectively. In unknown parameter case, each of systems has two unknown parameters and one known parameter. The stability of synchronization of three system is proved using Lyapunov stability theorem. Also, we use the synchronization of three chaotic systems with unknown parameters in secure communication via masking method. In secure communication, known parameters, unknown parameters, an affine combination of states, and coefficients are used for encryption and decryption. Numerical simulations are shown the effectiveness and feasibility of presented method.

Keywords: Lyapunov stability; Synchronization; Secure communication; Adaptive control; Nonlinear feedback

1 Introduction

Chaos, as an interesting phenomenon in nonlinear dynamical systems, has been studied widely over the last four decades [1–7]. Chaotic and hyperchaotic systems are nonlinear deterministic systems that display complex and unpredictable behavior. These systems have sensitivity with respect to initial conditions. The chaotic and hyperchaotic systems have many important fields in applied nonlinear sciences, such as laser physics, secure communications, nonlinear circuits, control, neural networks, and active wave propagation [2, 8–17].

Chaotic system synchronization has been investigated since Pecora and Carrol have introduced in 1990 [5]. Chaos synchronization, as an important topic in nonlinear science, has been widely investigated in many fields, such as physics, chemistry, ecological science, and secure communications [6, 18–20]. Various techniques have been proposed to achieve chaos synchronization such as adaptive control [21–23], active control [24], sliding-mode [25], and nonlinear control [26]. Recently, the synchronization of chaotic Complex systems was studied in [27, 28].

Secure communication was developed in 1992 based on synchronization of chaotic dynamical systems. The general idea for transmitting information via chaotic systems is that, an information signal is embedded in the transmitter system which produces a chaotic signal. The information signal is recovered by the chaotic receiver.

Chaotic communication techniques include chaos masking, chaos modulation, and chaos shift keying. In chaos masking the information signal is added directly to the transmitter. In chaos modulation is based on the masterslave synchronization, where the information signal is injected into the transmitter as a nonlinear filter. Chaos shift keying is supposed the information signal is binary is mapped into the transmitter and the receiver. In these three cases, the information signal can be recovered by a receiver if the transmitter and the receiver are synchronized [29–31].

In 1993, Cuomo et al. [32] developed the additive chaos masking approach. Dedieu et al. [33] presented the chaotic shift keying or the chaotic switching approach in 1993.

In 1996, Yang and Chua [34] introduced the chaotic parameter modulation method, where the information signal is used to modulate the chaotic systems parameters of in the transmitter.

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In 1997, Yang et al. [6] introduced a novel secure communication scheme by combining cryptography with chaos to generate what is called chaotic cryptrosystem. In this scheme, the information signal is encrypted by an encryption rule with a key generated from the chaotic system in the transmitter. Then, the techniques based on chaotic masking or chaotic modulation can be used to send the encrypted signal. At the receiver, a synchronization process between the two systems in the transmitter and the receiver is achieved and the signal is recovered. Then, the information signal can be retrieved by a decryption rule in the receiver.

The most of studies in synchronization and secure communications via chaotic systems are between two systems which are known drive and response systems. There are a few researches with three systems in this field [35–38].

In this paper, we study the synchronization of three chaotic systems with known and unknown parameters. In synchronization of three systems, we consider each system track each others, and each of systems are master and slave systems. In synchronization with unknown parameters, we let two unknown parameters for each systems. The T-system [39] is taken as an example to verify the results. We use this method for secure communications between three cases. Also, we use known parameters, unknown parameters, an affine combination of states, and coefficients of combination as codes for coding and decoding of signal. We use the feedback and adaptive control methods, for synchronization of three systems with known and unknown parameters, respectively. Stability of error dynamical systems and updating rules are obtained by Lyapunov theorem. Further, numerical simulations are computed to check the analytical expressions of controllers and estimation lows.

The rest of this paper is organized as follows: Section 2 briefly introduces the chaotic T-system. In Section 3, the synchronization of three chaotic T-systems and numerical simulations are addressed. In section 4, we propose a masking method for secure communication. Besides, the effectiveness of the proposed scheme is evaluated with simulations for continuous signal. Finally, concluding remarks are given in Section 5.

2 T-system

In 2005, Tigen [39] introduced a new real chaotic nonlinear system and called it T-system, as follows:

$$\begin{cases} \dot{x_1} = a(x_2 - x_1), \\ \dot{x_2} = (c-a)x_1 - ax_1x_3, \\ \dot{x_3} = x_1x_2 - bx_3, \end{cases} \tag{1}$$

where x_1, x_2, and x_3 are the state variables and a, b and c are real positive parameters. By choosing $a = 2.1$, $b = 0.6$, and $0 < c < 40$, the Lyapunov exponents in Fig. 1 shows that the system (1) is a chaotic system. Because one of the Lyapunov exponents is positive. Also it can be considered as a dissipative system, since sum of its Lyapunov exponents is negative. The attractors of chaotic systems are bounded but not a fixed point or limit cycle. It is a properties of chaotic systems [40]. Fig. 2 displays an attractor of the T-system for some parameters and initial conditions. Synchronization of this system can be used for cryptography and decryption of data in secure communication.

3 Synchronization of three systems

For synchronizing of three chaotic systems, assume the systems be defined as follows:

$$\dot{x} = f(x, p_1) + u, \tag{2}$$

$$\dot{y} = g(y, p_2) + v, \tag{3}$$

$$\dot{z} = h(z, p_3) + w, \tag{4}$$

where $x = (x_1, x_2, \cdots, x_n)^t, y = (y_1, y_2, \cdots, y_n)^t, z = (z_1, z_2, \cdots, z_n)^t \in R^n$ are the state vectors of the systems, $u, v,$ and w are n-dimensional control signals and $p_1, p_2,$ and p_3 are vector parameters (known or unknown). The goal is to design appropriate controllers $u, v,$ and w such that, for any initial conditions, we have

$$\lim_{t \longrightarrow \infty} x(t) = \lim_{t \longrightarrow \infty} y(t) = \lim_{t \longrightarrow \infty} z(t).$$

For this purpose, we define error dynamics as follows

$$\begin{cases} e_{xy} = x - y \quad or \quad e_{yx} = y - x, \\ e_{xz} = x - z \quad or \quad e_{zx} = z - x, \\ e_{yz} = y - z \quad or \quad e_{zy} = z - y. \end{cases} \tag{5}$$

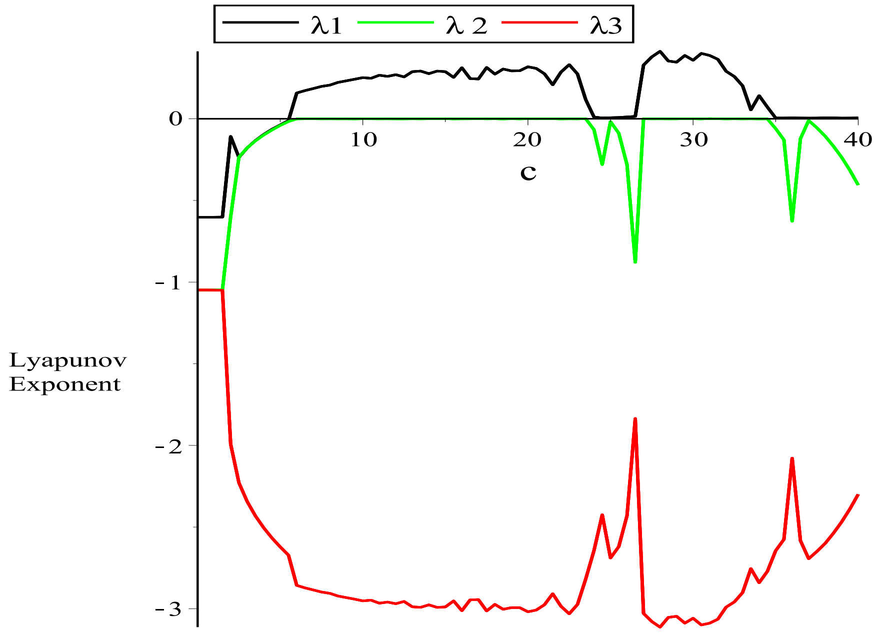

Figure 1: Lyapunov exponents of system (1), for $a = 2.1$, $b = 0.6$, and $0 < c < 40$.

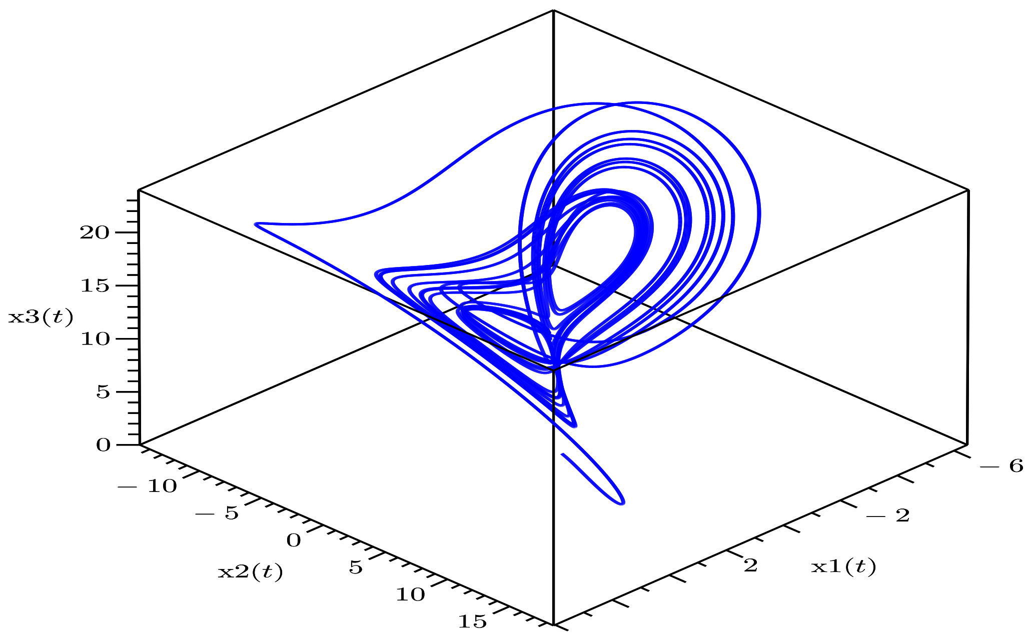

Figure 2: An attractor of T-system for $a = 2.1$, $b = 0.6$, and $c = 28$ with initial states $(x_1(0), x_2(0), x_3(0)) = (1, 3, 0)$.

Then it is sufficient we consider two of them. For this purpose, we take

$$\begin{cases} e_{xy} = x - y \rightarrow \dot{e_{xy}} = \dot{x} - \dot{y}, \\ e_{zx} = y - z \rightarrow \dot{e_{yz}} = \dot{y} - \dot{z}. \end{cases} \tag{6}$$

3.1 Synchronization of three system with known parameters

We consider three identical T-system with known parameters as follow

$$\begin{cases} \dot{x_1} = a(x_2 - x_1) + u_1, \\ \dot{x_2} = (c - a)x_1 - ax_1x_3 + u_2, \\ \dot{x_3} = x_1x_2 - bx_3 + u_3, \end{cases} \tag{7}$$

$$\begin{cases} \dot{y_1} = a(y_2 - y_1) + v_1, \\ \dot{y_2} = (c - a)y_1 - ay_1y_3 + v_2, \\ \dot{y_3} = y_1y_2 - by_3 + v_3, \end{cases} \tag{8}$$

and

$$\begin{cases} \dot{z_1} = a(z_2 - z_1) + w_1, \\ \dot{z_2} = (c - a)z_1 - az_1z_3 + w_2, \\ \dot{z_3} = z_1z_2 - bz_3 + w_3. \end{cases} \tag{9}$$

The errors among the three systems (7)-(9) are defined as follow

$$\begin{cases} e_1 = x_1 - y_1, \quad e_2 = x_2 - y_2, \quad e_3 = x_3 - y_3, \\ e_4 = x_1 - z_1, \quad e_5 = x_2 - z_2, \quad e_6 = x_3 - z_3, \\ e_7 = z_1 - y_1, \quad e_8 = z_2 - y_2, \quad e_9 = z_3 - y_3. \end{cases} \tag{10}$$

With regard to relations (5) and (6), we can use $e_1, e_2, \cdots, e_6$ of (10) for obtaining controllers. Then the error dynamical system is

$$\begin{cases} \dot{e_1} = a(e_2 - e_1) + u_1 - v_1, \\ \dot{e_2} = (c - a)e_1 - a(x_1x_3 - y_1y_3) + u_2 - v_2, \\ \dot{e_3} = x_1x_2 - y_1y_2 - be_3 + u_3 - v_3, \\ \dot{e_4} = a(e_5 - e_4) + u_1 - w_1, \\ \dot{e_5} = (c - a)e_4 - a(x_1x_3 - z_1z_3) + u_2 - w_2, \\ \dot{e_6} = x_1x_2 - z_1z_2 - be_6 + u_3 - w_3. \end{cases} \tag{11}$$

Theorem 1 *Systems (7)-(9) will be globally asymptotically synchronized for any initial condition with the following nonlinear feedback controller laws,*

$$\begin{cases} u_1 = u_2 = u_3 = 0, & \\ v_1 = ae_2, & w_1 = ae_5, \\ v_2 = (c - a)e_1 - a(x_1x_3 - y_1y_3) + e_2, & w_2 = (c - a)e_4 - a(y_1y_3 - z_1z_3) + e_5, \\ v_3 = x_1x_2 - y_1y_2 - be_3 + e_3, & w_3 = -z_1z_2 + x_1x_2 - be_6 + e_6. \end{cases} \tag{12}$$

Proof. *We define the Lyapunov function as follow:*

$$V(t) = \tfrac{1}{2} \textstyle\sum_1^6 e_i^2. \tag{13}$$

Using of (11) and (12) we have:

$$\dot{V(t)} = -\sum_1^6 e_i\dot{e}_i = -[a(e_1^2 + e_4^2) + (e_2^2 + e_3^2 + e_5^2 + e_6^2)] < 0.$$

It is clear that V is positive definite and $\dot{V}$ is negative definite. According to the Lyapunov stability theorem, the error system (11) converges to the equilibrium points of (11). It follow that the systems (7)-(9) synchronize asymptotically and globally. This completes the proof. ■

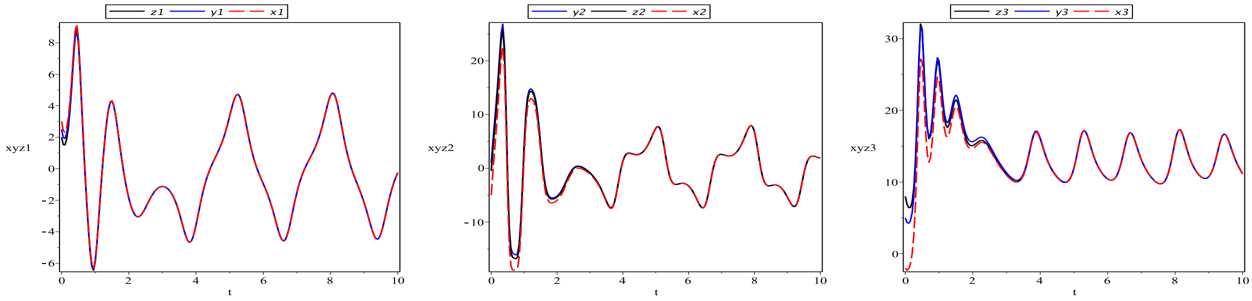

Figure 3: Synchronization of three systems with known parameters.

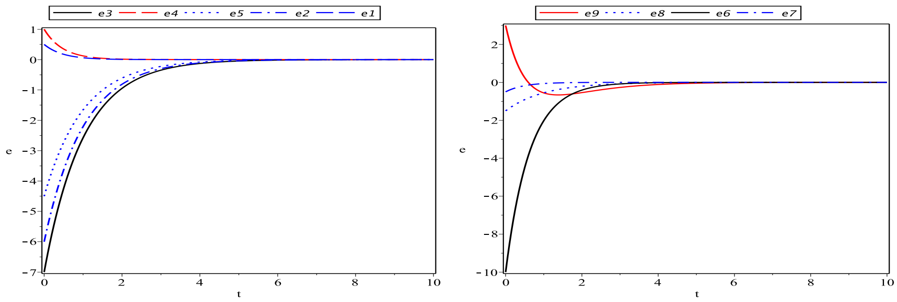

Figure 4: Errors due for synchronization of three systems with known parameters.

3.1.1 Numerical simulation for synchronization with known parameters

To demonstrate and verify the validity of the proposed scheme, we discuss and illustrate the numerical simulations results for chaotic T-system (1). Systems (7)-(9) with controllers (12) are solved numerically for $a = 2.1$, $b = 0.6$, and $c = 30$ with different initial conditions $(x_1(0), x_2(0), x_3(0)) = (3, -5, -2)$, $(y_1(0), y_2(0), y_3(0)) = (2.5, 1, 5)$, and $(z_1(0), z_2(0), z_3(0)) = (2, -0.5, 8)$. The results of chaotic synchronization of three identical chaotic T-systems via nonlinear feedback control is shown in Fig. 3. This shows the synchronization of (7)-(9) is achieved after small time. The errors due for synchronization are plotted in Fig. 4. As expected from the above analytical considerations the synchronization errors e_i converge to zero as $t \longrightarrow \infty$.

3.2 Synchronization of three system with unknown parameters

In this subsection, for synchronization of three systems, we assume that two parameters of systems is unknown. For instance, we consider

$$\begin{cases} \dot{x_1} = \hat{a}(x_2 - x_1) + u_1, \\ \dot{x_2} = (c - \hat{a})x_1 - \hat{a}x_1x_3 + u_2, \\ \dot{x_3} = x_1x_2 - \hat{b}x_3 + u_3, \end{cases} \tag{14}$$

$$\begin{cases} \dot{y_1} = \hat{a}(y_2 - y_1) + v_1, \\ \dot{y_2} = (\hat{c} - \hat{a})y_1 - \hat{a}y_1y_3 + v_2, \\ \dot{y_3} = y_1y_2 - by_3 + v_3, \end{cases} \tag{15}$$

and

$$\begin{cases} \dot{z_1} = a(z_2 - z_1) + w_1, \\ \dot{z_2} = (\hat{c} - a)z_1 - az_1z_3 + w_2, \\ \dot{z_3} = z_1z_2 - \hat{b}z_3 + w_3. \end{cases} \tag{16}$$

In (14)-(16), (a, b, c) and $(\hat{a}, \hat{b}, \hat{c})$ are known and unknown parameters respectively. Errors between the three systems (14)-(16) are defined as follow

$$\begin{cases} e_1 = x_1 - y_1, \quad e_2 = x_2 - y_2, \quad e_3 = x_3 - y_3, \\ e_4 = y_1 - z_1, \quad e_5 = y_2 - z_2, \quad e_6 = y_3 - z_3, \\ e_7 = x_1 - z_1, \quad e_8 = x_2 - z_2, \quad e_9 = x_3 - z_3. \end{cases} \tag{17}$$

As mentioned earlier, for synchronizing three proposed systems, it is sufficient the error system is considered as follows:

$$\begin{cases} \dot{e_1} = \hat{a}(e_2 - e_1) + u_1 - v_1, \\ \dot{e_2} = (\hat{c} - \hat{a})e_1 - \tilde{c}x_1 - \hat{a}(x_1x_3 - y_1y_3) + u_2 - v_2, \\ \dot{e_3} = x_1x_2 - y_1y_2 - \hat{b}e_3 - \tilde{b}y_3 + u_3 - v_3, \\ \dot{e_4} = \hat{a}(e_5 - e_4) + \tilde{a}(z_1 - z_2) + v_1 - w_1, \\ \dot{e_5} = (\hat{c} - \hat{a})e_4 - \hat{a}(y_1y_3 - z_1z_3) - \tilde{a}(z_1 + x_1z_3) + v_2 - w_2, \\ \dot{e_6} = y_1y_2 - z_1z_2 - \hat{b}e_6 + \tilde{b}y_3 + v_3 - w_3, \end{cases} \tag{18}$$

where $\tilde{a} = \hat{a} - a$, $\tilde{b} = \hat{b} - b$, and $\tilde{c} = \hat{c} - c$.

Theorem 2 *Systems (14)-(16) will be globally asymptotically synchronized for any initial condition with the following adaptive control and estimation laws*

$$\begin{cases} u_1 = u_2 = u_3 = 0, \\ v_1 = \hat{a}(e_2 - e_1) + e_1, \\ w_1 = v_1 + \hat{a}(e_5 - e_4) + e_4 = \hat{a}(e_8 - e_7) + e_7, \\ v_2 = (\hat{c} - \hat{a})e_1 - \hat{a}(x_1x_3 - y_1y_3) + e_2, \\ w_2 = v_2 + (\hat{c} - \hat{a})e_4 - \hat{a}(y_1y_3 - z_1z_3) + e_5 = (\hat{c} - \hat{a})e_7 - \hat{a}(x_1x_3 - z_1z_3) + e_8, \\ v_3 = x_1x_2 - y_1y_2 - \hat{b}e_3 + e_3, \\ w_3 = v_3 - z_1z_2 + y_1y_2 - \hat{b}e_6 + e_6 = x_1x_2 - z_1z_2 - \hat{b}e_9 + e_9, \end{cases} \tag{19}$$

$$\begin{cases} \dot{\tilde{a}} = \dot{\hat{a}} = -e_4(z_2 - z_1) + e_5z_1 + e_5z_1z_3, \\ \dot{\tilde{b}} = \dot{\hat{b}} = (e_3 - e_6)y_3, \\ \dot{\tilde{c}} = \dot{\hat{c}} = e_2x_1, \end{cases} \tag{20}$$

where $(\hat{a}, \hat{b}, \hat{c})$ is the estimates of (a, b, c).
Proof. *We define the Lyapunov function as follow:*

$$V(t) = \tfrac{1}{2}\textstyle\sum_{i=1}^{6} e_i^2 + \tfrac{1}{2}(\tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2). \tag{21}$$

Using (18) and (19) we have:

$$\dot{V(t)} = -\sum_{1}^{6} e_i^2 < 0.$$

It is clear that V is positive definite and $\dot{V}$ is negative definite. According to the Lyapunov stability theorem, the error system (18) and estimation rule (20) are convergence to the equilibrium points of (18) and (20). Therefore, systems (14)-(16) asymptotically and globally synchronized. ■

3.2.1 Numerical simulation for synchronization with unknown parameters

To demonstrate and verify the validity of the proposed scheme, we discuss and illustrate the numerical simulations results for chaotic T-system (1). Systems (14)-(16) and (20) with controllers (19) are solved numerically for $a = 2.1$, $b = 0.6$, and $c = 30$ with different initial conditions $(x_1(0), x_2(0), x_3(0)) = (-3, -5, -2)$, $(y_1(0), y_2(0), y_3(0)) = (2.5, 1, 5)$, $(z_1(0), z_2(0), z_3(0)) = (7, -0.5, 0)$, and the initial values of the parameters estimation laws are $(\hat{a}(0), \hat{b}(0), \hat{c}(0)) = (5, 0.8, 27)$. The results of chaotic synchronization of three identical chaotic T-systems via adaptive control is shown in Fig. 5. Synchronization of (14)-(16) is achieved after small time interval. The errors due of synchronization are plotted in Fig. 6. As expected from the above analytical considerations the synchronization errors e_i converge to zero as $t \longrightarrow \infty$. Fig. 7 shows the estimates $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$ of the unknown parameters, converge to $a = 2.1$, $b = 0.6$, and $c = 30$, respectively, as $t \longrightarrow \infty$.

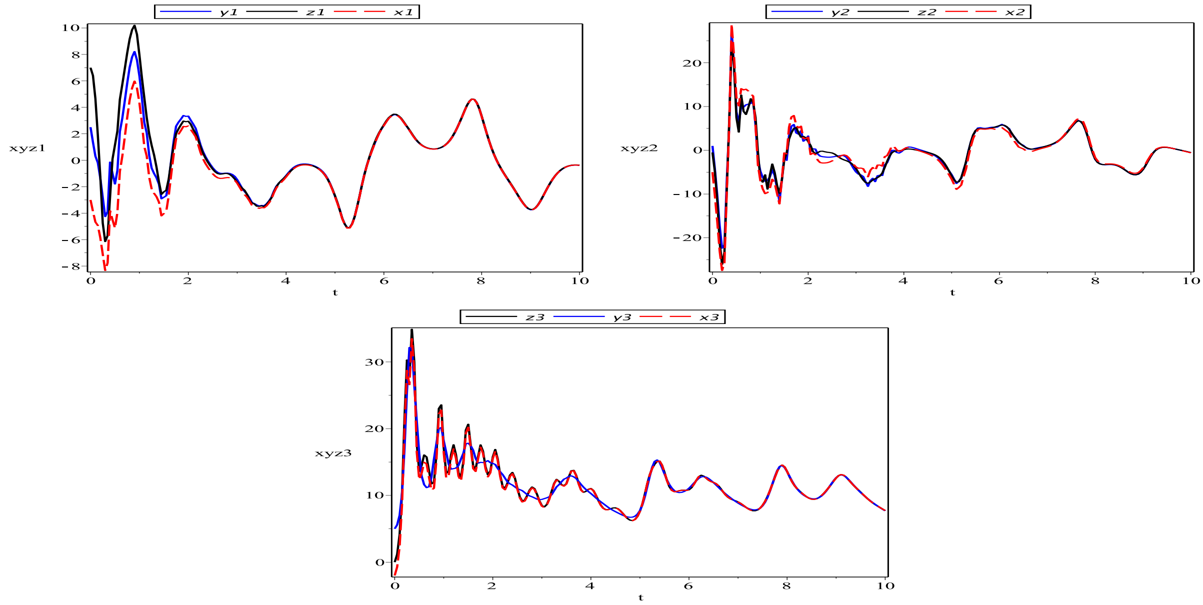

Figure 5: Synchronization of three systems with unknown parameters.

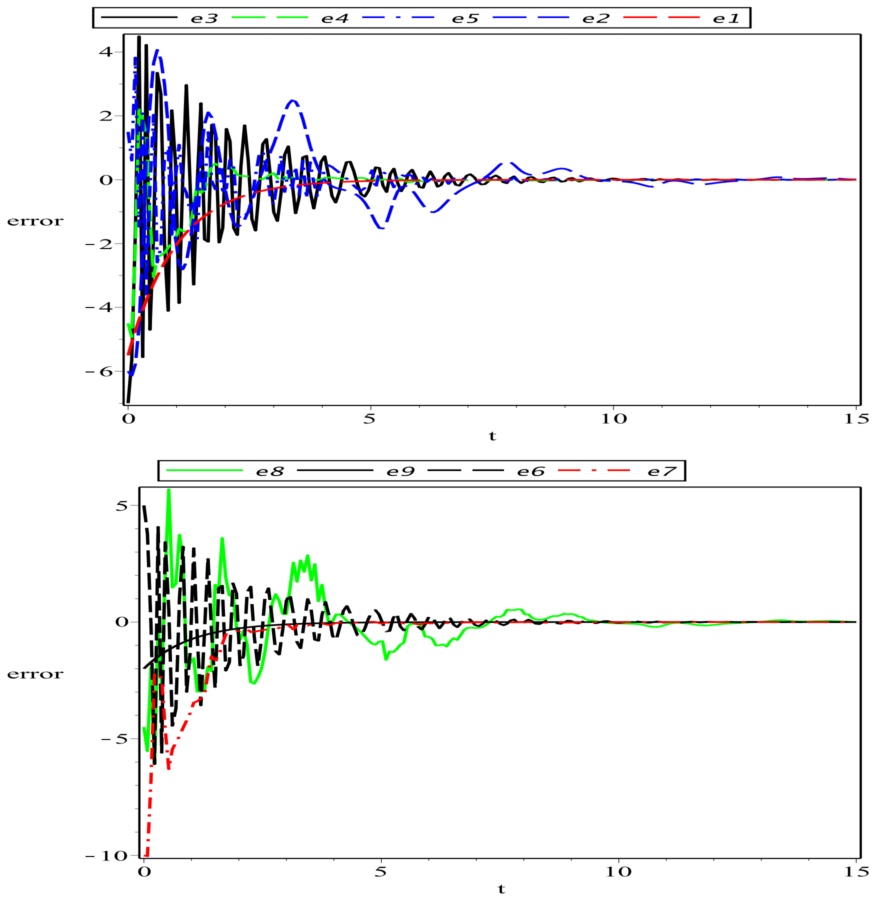

Figure 6: Errors due for synchronization of three system with unknown parameters.

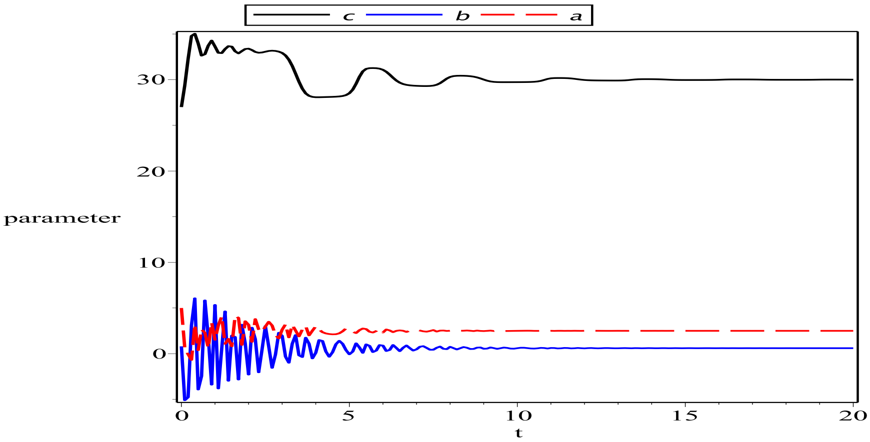

Figure 7: Estimation of unknown parameters.

4 Secure communication among three cases

In this section, we introduce a new secure communication between three case with ring connection via masking method. Assume that M_1, M_2, and M_3 are the message signals, such that carrier by systems I, II, and III respectively. Systems I, II, and III are designed according the systems (14), (15), and (16) with parameter estimation (20) and controllers (19). Fig. 8 depicts a block diagram of the of the proposed secure communication scheme based on three coupled T-system with known and unknown parameters. Systems are used as transmitter and receiver. Each of systems has a known parameter and two unknown parameters. When system is a transmitter, known parameter is used as a code. In two other systems, this parameter is unknown and will be estimated. The estimation of them are used for encoding in masking method. Also, we use ordinary encryption with chaotic encryption.

Let T_1, T_2, and T_3 are transmitted signals with systems I, II, and III respectively. Such that:

$$\begin{cases} T_1 = M_1 + c(k_{11}x_1 + k_{12}x_2 + k_{13}x_3), \\ T_2 = M_2 + b(k_{21}y_1 + k_{22}y_2 + k_{23}y_3), \\ T_3 = M_3 + a(k_{31}z_1 + k_{32}z_2 + k_{33}z_3), \end{cases} \tag{22}$$

where $\sum_{j=1}^{3} k_{ij} = 1, (i = 1, 2, 3)$ and $k_{ij} > 0$. Parameters and k_{ij} are used as codes. These signals are encrypted by chaos. By noting the synchronization of three coupled systems, we decrypt these signals. For this aim, let

$$\begin{cases} R_{12} = T_1 - \hat{c}(k_{11}y_1 + k_{12}y_2 + k_{13}y_3), & R_{13} = T_1 - \hat{c}(k_{11}z_1 + k_{12}z_2 + k_{13}z_3), \\ R_{21} = T_2 - \hat{b}(k_{21}x_1 + k_{22}x_2 + k_{23}x_3), & R_{23} = T_2 - \hat{b}(k_{21}z_1 + k_{22}z_2 + k_{23}z_3), \\ R_{31} = T_3 - \hat{a}(k_{31}x_1 + k_{32}x_2 + k_{33}x_3), & R_{32} = T_3 - \hat{a}(k_{31}y_1 + k_{32}y_2 + k_{33}y_3), \end{cases} \tag{23}$$

where R_{ij} $(i, j = 1, 2, 3)$ show the recovered signals by system j such that, it transferred by system i. $\hat{a}, \hat{b}$, and $\hat{c}$ are unknown parameters, such that estimated by parameters estimation rule.

To demonstrate and verify the validity of the proposed scheme, we present and discuss the numerical simulations results to encrypt and decrypt of signals. Let

$$\begin{cases} M_1 = 5|[\sin(2\pi t)]|, \\ M_2 = 5|[\cos(3\pi t)]|, \\ M_3 = 5|[\cos(\pi t) + \sin(\pi t)]|, \end{cases} \tag{24}$$

where $[.]$ and $|.|$ are integrate part and absolute value functions respectively. Also, let $K = [K_{ij}]$ consider as follow

$$K = 0.1 \begin{bmatrix} 6 & 3 & 1 \\ 5 & 3 & 2 \\ 2 & 3 & 5 \end{bmatrix}. \tag{25}$$

The result of presented method is shown in Figs. (9)-(11).

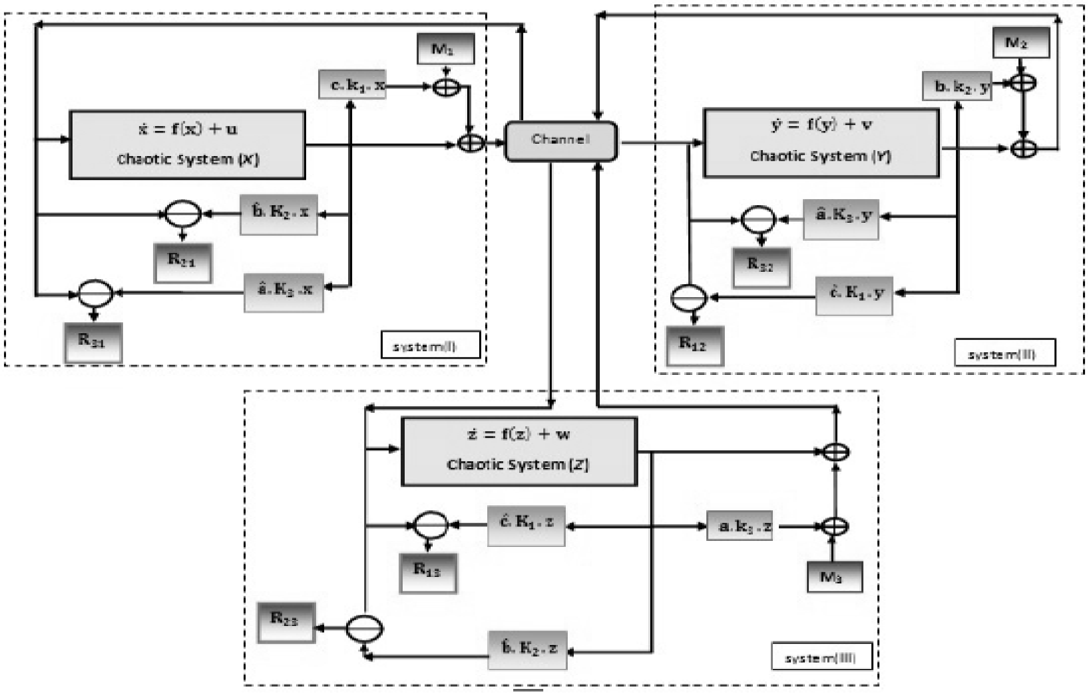

Figure 8: Diagram of secure communication among three cases, where $k_1 = (k_{11}, k_{12}, k_{13})$, $k_2 = (k_{21}, k_{22}, k_{23})$, $k_3 = (k_{31}, k_{32}, k_{33})$ $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$, and $z = (z_1, z_2, z_3)$.

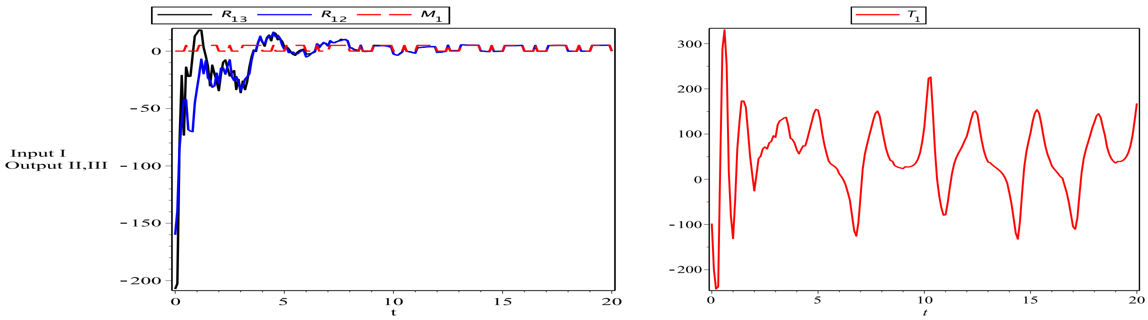

Figure 9: M_1 is massage signal that carried by system I, R_{12} and R_{13} are recovered signals by systems II and III, and T_1 is transmitted signal from I to II and III.

5 Conclusions

In this paper, we studied the synchronization of three identical chaotic T-system with known and unknown parameters via nonlinear feedback and adaptive control respectively. Control laws and parameter estimation rules were satisfied in Lyapunov stability theorem. Three chaotic systems with known and unknown parameters could be synchronized successfully. The result of synchronization coupled chaotic systems were used for secure communications among three systems via masking method. In used method to secure communication, known parameters, unknown parameters, affine combination of states, and coefficients were codes for coding and decoding of signal. In this method chaotic and ordinary encryption and decryption were used in secure communication. Numerical simulations were shown the feasibility of analytical predictions.

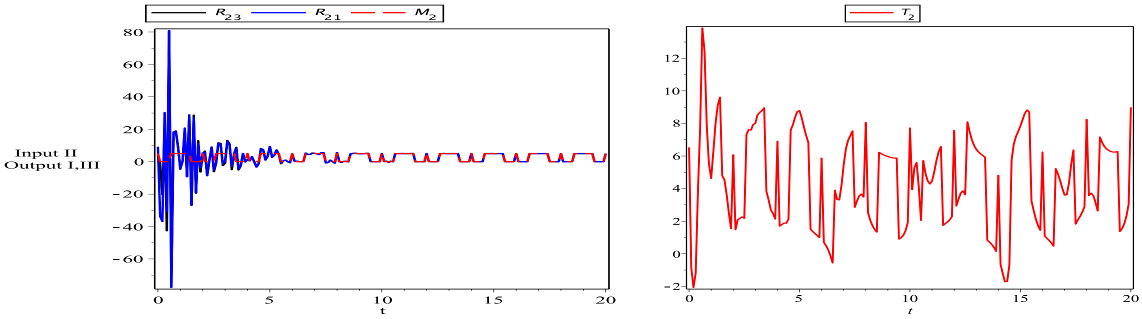

Figure 10: M_2 is massage Signal carried by system II, R_{21} and R_{23} are recovered signals by systems I and III, and T_2 is transmitted signal from II to I and III.

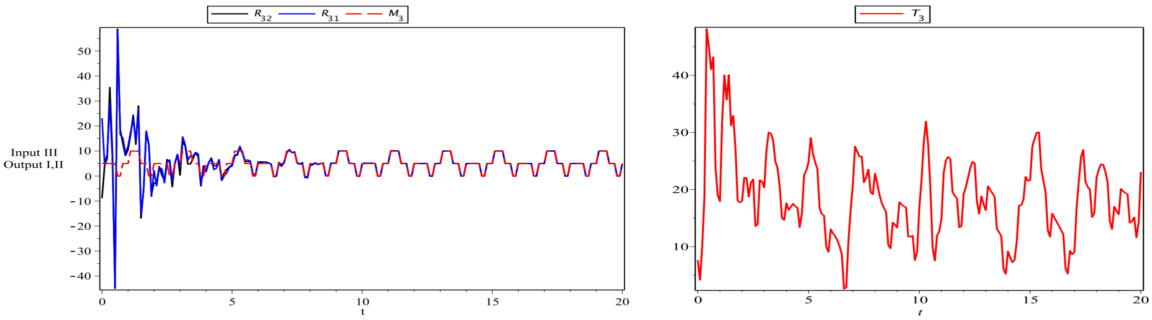

Figure 11: M_3 is massage Signal carried by system III, R_{31} and R_{32} are recovered signals by systems I and II, and T_3 is transmitted signal from III to I and II.

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