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Weighted Knowledge Diffusion Model

Yanan Gao[1], Lixin Tian[1,2]*
[1]Energy Development and Environmental Protection Strategy Research Center, Jiangsu University, Zhenjiang, Jiangsu 212013, China
[2]Energy Interdependency Behavior and Strategy Research Center, School of Mathematical Science, Nanjing Normal University, Nanjing, Jiangsu 210042, China
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Abstract: Based on the differences in the weights of the edges, In this paper, we establish a weighted knowledge diffusion model. Specifically, we work out the basic reproduction number regarded as the criterion of the knowledge diffusion, and perform representative numerical simulations. The results show that when the basic reproduction number is greater than 1, the knowledge eventually diffusion in the network; when the basic reproduction number is less than 1, the knowledge eventually fail to diffusion. At the same time, we find that the weighted knowledge diffusion network model more reflects the reality of the knowledge diffusion mechanism.

Keywords: knowledge diffusion; weighting factor; the basic reproduction number

1 Introduction

In today's society, knowledge is recognized as the driver of productivity and economic growth [1]. In a knowledge diffusion process, people can communicate what they know to others, or learn what others know [2]. The knowledge diffusion process results in sharing of knowledge assets by a large number of members [3].

Based on the analogy between epidemic spreading and knowledge diffusion, many researchers have been extending epidemic spreading models to research knowledge diffusion models [4][5]. Some researchers has been conducted using dynamical systems theory to reveal the effect of different mechanisms on knowledge diffusion [6][7][8][9][10][11][12]. Bin et al. considered the influence of the degree of forgetfulness on knowledge diffusion, established a knowledge diffusion model, and analyzed the stability of the equilibrium point and diffusion threshold [13]. Knowledge diffusion is different from epidemic spreading. People who have forgotten knowledge can reacquire it by review. If one person has not reviewed learned knowledge for a long time, memory retention will gradually decrease over time [14][15]. Some researchers have established a Knowledge diffusion model about the self-learning rate and the reviewing rate, which has a positive impact on knowledge transmission[16]. However, in the real world, the topological structure of the knowledge diffusion network is generally non-uniform, and most of them are scale-free, and this non-uniformity is not only reflected in the number of connected edges, but also reflected in the difference in edge weights [17][18][19][20]. Many previous researchers have not considered the weight of the edge of the network [4][5][16]. So in order to explore the mechanism of knowledge diffusion, this paper takes into account the differences in the weights of connected edges, and establishes a weighted knowledge diffusion model.

The rest of this paper is organized as follows. In Section 2, we introduced the diffusion mechanism of the knowledge diffusion model. In Section 3, we introduced the weighted knowledge diffusion model, and used the mean field theory to solve the basic reproduction number in heterogeneous networks. In Section 4, numerical simulations are performed to investigate theoretical. In Section 5, we fond that the final knowers density decreases almost linearly with the increase of the weighting factor, which indicates that the weighted knowledge diffusion network model more reflects the actual knowledge diffusion mechanism.

*Corrsponding author: Lixin Tian. *E-mail address*: tianlx@ujs.edu.cn

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2 Knowledge diffusion process

The study of knowledge diffusion is an important application of complex network theory. It is assumed that the network has N nodes, knowledge is propagated among the N nodes, and the edge is the communication channel between the two individuals. In addition, we also assume that the network is undirected. The population is divided into three categories: knowers, ignorants and forgetters. Knowers (I), are those who have acquired the knowledge; ignorants (S) are those people who do not have the knowledge, but may turn into knowers by contact with knowers or by self-learning; forgetters (N) are those who had the knowledge recently but forgot it. We assume that all of the individuals are willing to acquire knowledge and that knowers can share the knowledge to their neighbors. According to the above knowledge transmission rules, the knowledge diffusion rules can be expressed as follows[16]:

(1) Knowledge sharing. Ignorants can acquire knowledge by making a connection with knowers, and become knowers with probability λ. That is, λ denotes the diffusion rate.

(2) Self-learning. Let the self-learning rate δ reflect people's mean incorporated self-learning ability. Ignorants become knowers by self-learning with probability $\delta S_i(t)I\left(S_i(t)\right)$, where $I\left(S_i(t)\right)$ represents the fraction of neighbors in knowers i at time t [4][16].

(3) Forgetting. Because people do not always remember previously learned knowledge, knowers can lose knowledge over time. In this paper, we assume that all knowers forget at an identical rate μ. That is, μ is the forgetting rate.

(4) Reviewing. Forgetters can recover the knowledge by reviewing it. Let ξ , where $0 \leq \xi \leq 1$, denotes the mean reviewing rate [16].

Then, the knowledge diffusion model can be established according to the above rules, and the process of knowledge diffusion as shown in Fig. 1.

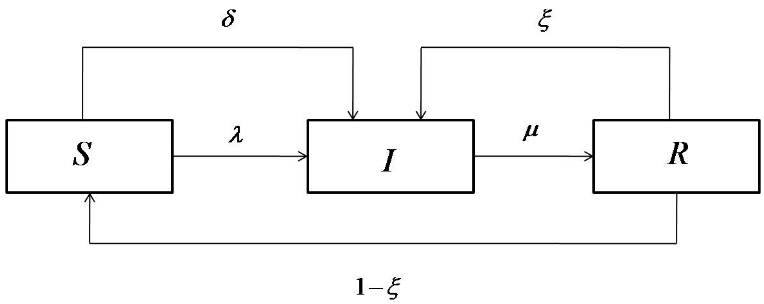

Figure 1: Process of knowledge diffusion

3 Weighted knowledge diffusion model

The topological structure of knowledge diffusion networks in the real world is generally non-uniform, and most of them are scale-free. This non-uniformity is not only reflected in the number of connected edges, but also in the weight of edges. In a knowledge transmission network, the weight of the connected edges on the network often indicates the closeness of the connected nodes or the length of the contact time, which can be used to measure the probability of knowledge transmission. The greater the weight, the more the two nodes connected intimate, and the greater the probability of spreading [17][18][19][20].

In this paper, the density of ignorants is denoted by $S_k(t)$ with degree k at time t, the density of knowers is denoted by $I_k(t)$ with degree k at time t, and the density of forgetters is denoted by $R_k(t)$ with degree k at time t.

Further, based on the rules of knowledge diffusion and the state transition process of nodes, a weighted knowledge diffusion model is established as follows:

$$\left\{\begin{array}{l} \frac{dS_k(t)}{dt} = (1-\xi)R_k(t) - \delta S_k(t)\sum_{k'} P\left(k'|k\right) I_{k'}(t)\lambda_{k'k} - kS_k(t)\sum_{k'} P\left(k'|k\right) I_{k'}(t)\lambda_{k'k} \\ \frac{dI_k(t)}{dt} = kS_k(t)\sum_{k'} P\left(k'|k\right) I_{k'}(t)\lambda_{k'k} + \delta S_k(t)\sum_{k'} P\left(k'|k\right) I_{k'}(t)\lambda_{k'k} + \xi R_k(t) - \mu I_k(t) \\ \frac{dR_k(t)}{dt} = \mu I_k(t) - R_k(t) \end{array}\right. , \quad (1)$$

where $\lambda_{k'k}$ is the probability that an ignorant node with a degree of k is connected to a knower node with a degree of k' and is spread, $w_{k'k} = \mathrm{w}_0\left(k'k\right)^{\eta}$is the edge weight of a node with degree k connected to a node with degree k', where η

is called weighting factor. The propagation rate of a node is related to weight of the edges, and the greater the weight, the more likely it is that the node will be propagated through this edge, so $\lambda_{k',k} = \lambda k \frac{w_{k'k}}{N_k}$, $N_k = k\sum_{k'} P(k'|k)\, w_{k'k}$. So Eq.(1) can be transformed into

$$\begin{cases} \frac{dS_k(t)}{dt} = (1-\xi)R_k(t) - \delta S_k(t) < k > \Theta(t) - \frac{k^{1+\eta}<k>}{<k^{1+\eta}>}\lambda S_k(t)\Theta(t) \\ \frac{dI_k(t)}{dt} = \frac{k^{1+\eta}<k>}{<k^{1+\eta}>}\lambda S_k(t)\Theta(t) + \delta S_k(t) < k > \Theta(t) + \xi R_k(t) - \mu I_k(t) \\ \frac{dR_k(t)}{dt} = \mu I_k(t) - R_k(t) \end{cases}, \tag{2}$$

where $\Theta(t)$ represents the probability with which a knower individual could get in touch with an Ignorants. In the uncorrelated networks, $\Theta(t)$ can be written as $\Theta(t) = \frac{1}{<k>}\sum_k kP(k)I_k(t), k = k_{\min}, \ldots, k_{\max}$, and $< k >= \sum_k kP(k)$ is the average degree and $p(k)$ is the degree distribution of the network.

Letting $\frac{dS_k(t)}{dt} = 0$, $\frac{dI_k(t)}{dt} = 0$, $\frac{dR_k(t)}{dt} = 0$, we obtain the knowledge-propagating equilibrium point (S_k^*, I_k^*, R_k^*), it has the following form:

$$\begin{cases} S_k^* = \frac{\mu(1-\xi)}{\mu(1-\xi)+(1+\mu)\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>}+\delta\right)<k>\Theta} \\ I_k^* = \frac{\left(\lambda\frac{k^{1+\eta}}{\langle k^{1+\eta}\rangle}+\delta\right)<k>\Theta}{\mu(1-\xi)+(1+\mu)\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>}+\delta\right)<k>\Theta} \\ R_k^* = \frac{\mu\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>}+\delta\right)<k>\Theta}{\mu(1-\xi)+(1+\mu)\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>}+\delta\right)<k>\Theta} \end{cases}. \tag{3}$$

Substituting I_k^* into $\Theta(t)$, we obtain the self-consistency equality

$$\begin{aligned} \Theta(t) &= \frac{1}{< k >}\sum_k kP(k)I_k(t) \\ &= \frac{1}{< k >}\sum_k kP(k)\frac{\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>} + \delta\right) < k > \Theta}{\mu(1-\xi) + (1+\mu)\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>} + \delta\right) < k > \Theta}. \end{aligned}$$

Define

$$f(\Theta) = \Theta - \frac{1}{< k >}\sum_k kP(k)\frac{\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>} + \delta\right) < k > \Theta}{\mu(1-\xi) + (1+\mu)\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>} + \delta\right) < k > \Theta}. \tag{4}$$

It is obvious that $\Theta = 0$ is a solution to Eq.(4). What we are interested in is the conditions under which a nontrivial solution to Eq.(4) exists. Since $0 < f(1) \leq 1$, to ensure a nontrivial solution $\Theta \in (0, 1]$, it suffices to satisfy the inequality

$$\begin{aligned} \left.\frac{df(\Theta)}{d\Theta}\right|_{\Theta=0} &= 1 - \frac{1}{< k >}\sum_k kP(k)\frac{\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>} + \delta\right) < k >}{\mu(1-\xi)} \\ &= 1 - R_0 < 0, \end{aligned}$$

where R_0 is given by

$$R_0 = \frac{1}{\mu(1-\xi)}\left(\frac{\lambda < k^{2+\eta} >}{< k^{1+\eta} >} + \delta < k >\right).$$

Therefore, when $R_0 > 1$, there exists at least one endemic equilibrium. In addition, we can get the second derivative of $f(\Theta)$, i.e.,

$$\frac{df^2(\Theta)}{d\Theta^2} = \frac{1}{< k >}\sum_k kP(k)\frac{2\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>} + \delta\right)^2 \mu(1-\xi)(1+\mu) < k >^2}{\left(\mu(1-\xi) + (1+\mu)\left(\lambda\frac{k^{1+\eta}}{<k^{1+\eta}>} + \delta\right) < k > \Theta\right)^3}.$$

Since $0 \leq \xi \leq 1$, it is obvious that $f(\Theta)\frac{df^2(\Theta)}{d\Theta^2} \geq 0$. Then $f(\Theta)$ is a convex function in $\Theta \in [0, 1]$. Hence, if $R_0 < 1$, $f(\Theta)$ has no solution in (0,1]. However, if $R_0 > 1$ then there exists an unique solution.

4 Simulation and discussion

In this section, we use numerical simulations to verify the theoretical derivation and test the sensitivity of the proposed model to relevant parameters. To explore the diffusion of knowledge in heterogeneous networks, we study the representative networks (BA scale-free networks). In the network, the total population is set to $N = 2000$. In addition, in order to avoid random effects, all simulation results were averaged over 100 independent runs.

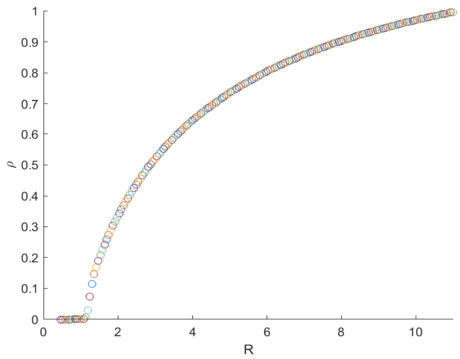

Figure 2: The relationship between the basic reproduction number and the final knowers density in the BA network

Fig.2 shows the relationship between the basic reproduction number and the final knowers density in the BA model. It can be found that when the basic reproduction number is greater than 1, the knowers will propagate in the network and tend to be stable with time; when the basic reproduction number is less than 1, the knowers density gradually decreases with time, and eventually tends to zero, which means that knowledge will not spread in the network overtime.

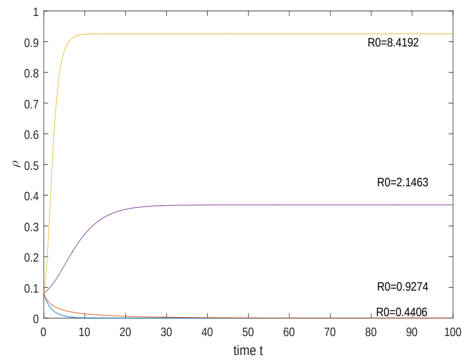

Figure 3: The knowers density changes with time in the BA network

In order to verify the conclusion in Fig.2 again, that is, in a non-uniform network, when the basic reproduction number is greater than 1, the knowers will propagate in the network and tend to be stable with time; when the basic reproduction number is less than 1, the knowers density gradually decreases with time, and eventually tends to zero. Here, different parameters are taken to draw the change of the knowers density in heterogeneous network with time. Fig.3 shows the results of the knowers density changes over time when the basic reproduction number are different, and the result is consistent with the conclusion obtained in Fig.2.

Fig.4 shows the relationship between the final density of knowers and the weighting factor when other parameters are constant. We can find that the density of knowers decreases with the increase of the weighting factor, and the density of knowers decreases almost linearly with the increase of the weighting factor. Because a heterogeneous network has serious heterogeneity, very little nodes in the network have extremely many connections, and the weight of the connected edges is large, but most nodes have only a very little connections, and the edge weights are small, which will affect the diffusion of knowledge. Therefore, compared with the general knowledge diffusion network model, the weighted knowledge diffusion network model reflects the actual knowledge diffusion mechanism more.

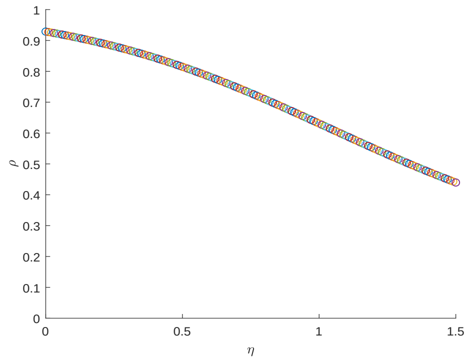

Figure 4: The relationship between the final density of knowers and the weighting factor in the BA network

5 Conclusions

In order to explore the mechanism of knowledge diffusion, based on the differences in the weights of the edges in real networks, this paper establish a weighted knowledge diffusion model. We use the mean field theory to solve the basic reproduction number of the knowledge diffusion model in heterogeneous networks, to verify the theoretical analysis, we perform a series of numerical simulations, and the results clearly show that there is excellent consistency between the theoretical derivation and the numerical simulations in terms of the basic reproduction number.

We can draw conclusions that when the basic reproduction number is greater than 1, the knowledge will eventually diffusion in the network; when the basic reproduction number is less than 1, the knowledge will eventually fail to diffusion in the network. At the same time, we find that the density of the final knowers decreases almost linearly with the increase of the weighting factor, which means the weighted knowledge diffusion network model reflects the actual knowledge diffusion mechanism more. Therefore, in real life, people should increase communication with each other, which will further promote the diffusion of knowledge.

Acknowledgments

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