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A New Algorithm for Fuzzy Linear Regression with Crisp Inputs and Fuzzy Output

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Abstract: In this work, the parameters of fuzzy linear regression based on the least squares approach is computed by ST-decomposition method. This method is not an iterative technique, however, it is a powerful method for nonsingular coefficient matrices. Numerical examples are at the end of this paper to illustrate the performance of the new method.

Keywords: fuzzy linear regression; least square; ST-decomposition; fuzzy number

1 Introduction

Regression analysis is one of the useful computational issues of Statistics and Mathematics that most of researchers even in other fields such as economic, engineering, social science etc. may need the estimation of their observations by an appropriate regression model. Among different regression methods linear regression is more popular, which mainly the regression parameters of this method are computed by a least square problem. However, classical regression analysis may not be able to give an appropriate model for all kinds of observation data set especially for the problems with vague variables. Therefore, fuzzy linear regression, which is much more flexible for different problems rather than classical regression, was proposed by Tanaka et al. [1] in 1982. Later on, many investigations have focused on different variants of fuzzy regression, their properties and applications ([2-6]). Generally, all approaches for fuzzy linear regression can be divided into three main categories of fuzzy linear regression based on minimizing the fuzziness, fuzzy least squares regression and fuzzy regression based on interval analysis [6].

In this paper, a fuzzy linear regression (FLR) based on least squares technique of Diamond [5] and its updated version in matrix form [7] is considered. This method is selected for estimating the solution of fuzzy linear regression for a set of observations (x_i, y_i) , $i = 1, 2, \ldots, n$ with crisp numbers x_i (inputs) and triangular fuzzy numbers y_i (outputs) because it has some more benefits rather than other variants [6-8].There are many investigations regarding to different variants of Fuzzy Linear Regression as well as the applications of such fuzzy regressions in science and technology that some of the newest published papers in this area are mentioned here for the interested readers who like to extend their knowledge [14-25]. In [14-16] there are some techniques for estimating the solution of different fuzzy linear regressions. Least square approach is so popular among researchers even for fuzzy regression problems. For example, Nong in [17] and Wu in [19] discussed some methods and notes for fuzzy least squares, D'Urso and Massari proposed an iterative weighted least square approach for such regressions in [18] and Yongqi [20] and Chaudhuri [21] proposed some Least Square approaches for Fuzzy Support Vector Regression. Shakouri and Nadimi [22] suggested a fuzzy regression method for detection of outlier data to eliminate or reduce the influence of such data (which normally leads to inaccurate results). Pourahmad and her colleagues [23] suggested a fuzzy regression method using the least square approach for their clinical study. Chung in [24] used fuzzy linear regression for to optimize the benchmarking system for energy efficiency of commercial buildings. In [25], Chan and his colleagues used a fuzzy regression model to support products for Customer Satisfaction. The outline of this paper is as follows. In section 2, the primary notations and definitions of fuzzy numbers and FLR are discussed. In section 3, a general form of FLR is considered and a new iterative method for estimating the solution of such a problem is argued. Numerical experiment is in section 4 to examine the performance of the new method. Finally, conclusion is the last sections of this paper.

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2 Preliminary definitions

In this section, some primary definitions and notes, which are required in this work, have been written [9, 10].

Definition 1 *A fuzzy set $\tilde{u}$, defined as a map from $\mathbb{R}$ into $[0,\ 1]$, is a fuzzy number if its membership function $\mu_{\tilde{u}}$ satisfy the following conditions:*

1. *$\tilde{u}$ is convex; i.e.$\mu_{\tilde{u}}\left(\lambda x_1 + (1-\lambda)\, x_2\right) \geq \min\left\{\mu_{\tilde{u}}\left(x_1\right),\ \mu_{\tilde{u}}\left(x_2\right)\right\}$, $\forall x_1, x_2 \in \mathbb{R}$ and $\forall \lambda \in [0,1]$.*
2. *$\exists x_0 \in [0,1] :\ \mu_{\tilde{u}}\left(x_0\right) = 1$,*
3. *$\mu_{\tilde{u}}$ is a piecewise continuous function.*

Definition 2 *A fuzzy number $\tilde{u}$ is represented by $\tilde{u}\left(r\right) = \left(\underline{u}\left(r\right), \bar{u}\left(r\right)\right)$ where $\underline{u}\left(r\right)$ and $\bar{u}\left(r\right)$ are two functions on $[0,1]$ which satisfy the following requirements:*

1. *$\underline{u}$ is a bounded monotonically increasing left continuous function.*
2. *$\bar{u}$ is a bounded monotonically decreasing left continuous function.*
3. *$\underline{u}\left(r\right) \leq \bar{u}\left(r\right),\ \ \forall r \in [0,1]$.*

Definition 3 *A crisp number α can simply be represented by $\underline{u}\left(r\right) = \bar{u}\left(r\right) = \alpha,\ \ \forall r \in [0,1]$.*

Definition 4 *For two arbitrary fuzzy numbers $\tilde{u}\left(r\right) = \left(\underline{u}\left(r\right), \bar{u}\left(r\right)\right)$, $\tilde{v}\left(r\right) = \left(\underline{v}\left(r\right), \bar{v}\left(r\right)\right)$ and a crisp number α, the following operations are defined:*

1. *$\tilde{u} = \tilde{v}$ iff $\underline{u}\left(r\right) = \underline{v}\left(r\right)$, $\bar{u}\left(r\right) = \bar{v}\left(r\right)$, $\forall r \in [0,1]$,*
2. *$\tilde{u}\left(r\right) \pm \tilde{v}\left(r\right) = \left(\underline{u}\left(r\right) \pm \underline{v}\left(r\right), \bar{u}\left(r\right) \pm \bar{v}\left(r\right)\right)$,*
3. *$\alpha\, \tilde{u} = \begin{cases} \left(\alpha\, \underline{u}, \alpha\, \bar{u}\right), & \alpha \geq 0 \\ \left(\alpha\, \bar{u},\ \alpha\, \underline{u}\right), & \alpha < 0 \end{cases}$*

Definition 5 *A fuzzy number $\tilde{u}$ with the membership function*

$$\tilde{u} = \begin{cases} \frac{x-u_1}{u_2-u_1}, & u_1 \leq x < u_2 \\ \frac{u_3-x}{u_3-u_2}, & u_2 \leq x \leq u_3 \\ 0, & O.W. \end{cases} \tag{1}$$

is a triangular fuzzy number represented by $\tilde{u} = (u_1, u_2, u_3)$, which its parametric form is $\tilde{u}\left(r\right) = \left(\underline{u}\left(r\right), \bar{u}\left(r\right)\right)$ where $\underline{u}\left(r\right) = \left(u_2 - u_1\right) r + u_1$ and $\bar{u}\left(r\right) = u_3 - \left(u_3 - u_2\right) r$.

Definition 6 *A fuzzy number $\tilde{u}$is a nonnegative fuzzy number iff $\tilde{u}\left(r\right) = 0,\ \ \forall r < 0$.*

Definition 7 *A $m \times n$ linear system of equations*

$$\begin{array}{l} a_{1,1}\tilde{x}_1 + a_{1,2}\tilde{x}_2 + \cdots + a_{1,n}\tilde{x}_n = \tilde{y}_1, \\ a_{2,1}\tilde{x}_1 + a_{2,2}\tilde{x}_2 + \cdots + a_{2,n}\tilde{x}_n = \tilde{y}_2, \\ \ \vdots \\ a_{m,1}\tilde{x}_1 + a_{m,2}\tilde{x}_2 + \cdots + a_{m,n}\tilde{x}_n = \tilde{y}_m. \end{array} \tag{2}$$

is called a fuzzy linear system (FLS) where the elements of the $m \times n$ coefficient matrix $A = \left(a_{i,j}\right)$ are crisp numbers and $\tilde{y}_i,\ \ 1 \leq i \leq m$are fuzzy numbers.

Definition 8 *A fuzzy vector $x = \left(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n\right)^T$given by $\tilde{x}_i = \left(\underline{x}_i, \bar{x}_i\right),\ \ 1 \leq i \leq n$, in parametric form, is called a solution of fuzzy linear system iff*

$$\sum_{j=1}^{n} \underline{a_{i,j} x_j} = \underline{y}_i \quad , \quad \sum_{j=1}^{n} \overline{a_{i,j} x_j} = \bar{y}_i\,. \tag{3}$$

For solving (2)*, Friedman et al. [10] extended FLS* (2) *to a* $2m \times 2n$ *crisp linear system:*

$$BX = Y, \tag{4}$$

where the structure of $B = (b_{k,l})$ *and vectors* X, Y *are as:*

$$\begin{cases} b_{i,j} = b_{i+n,j+n} = a_{i,j}, & if\ a_{i,j} \geq 0, \\ b_{i+n,j} = b_{i,j+n} = -a_{i,j}, & if\ a_{i,j} < 0, \\ b_{i,j} = 0, & O.W. \end{cases}$$

$$X = (\underline{x}_1, \ldots, \underline{x}_n, -\bar{x}_1, \ldots, -\bar{x}_n)^T, Y = \left(\underline{y}_1, \ldots, \underline{y}_m, -\bar{y}_1, \ldots, -\bar{y}_m\right)^T.$$

In matrix form, we have

$$B = \begin{pmatrix} M & N \\ N & M \end{pmatrix},$$

where $M, N \geq 0$ *and* $A = M - N$*. So,* (4) *can be written as*

$$\begin{pmatrix} M & N \\ N & M \end{pmatrix} \begin{pmatrix} \underline{X} \\ -\bar{X} \end{pmatrix} = \begin{pmatrix} \underline{Y} \\ -\bar{Y} \end{pmatrix}.$$

In [10], the following theorems proved regarding to FLS (4) *and matrix* B.

1. *The matrix* B *is nonsingular iff both of* $A = M - N$ *and* $M + N$ *are nonsingular.*
2. *The structure of* B^{-1} *is the same as of* S*, i.e.* $B^{-1} = \begin{pmatrix} P & Q \\ Q & P \end{pmatrix}$.

3 Fuzzy Linear Regression

Now, a general form of a fuzzy linear regression is represented by

$$\tilde{y} = \tilde{a}_0 + \tilde{a}_1 x_1 + \tilde{a}_2 x_2 + \cdots + \tilde{a}_n x_n, \tag{5}$$

where $x_i,\ 1 \leq i \leq n$ are crisp independent variables (inputs), $\tilde{a}_j,\ 0 \leq j \leq n$ are fuzzy numbers as the regression coefficients(unknown parameters), $\tilde{y}$ is mainly a fuzzy dependent variable (output).

The main aim of fuzzy regression techniques is to find a model with the least estimation error which fits the best with observations. In current fuzzy least square approach, the FLR (5) is usually obtained by computing the regression parameters $\tilde{a}_i,\ 1 \leq i \leq n$ based on an observed data set $(X_i, \tilde{y}_i)$ where $X_i = (1, x_1, x_2, \ldots, x_n)^T$ so that $X_i^T \tilde{a} = \tilde{y}_i$ i.e.

$$\tilde{y}_i = \tilde{a}_0 + \tilde{a}_1 x_{i,1} + \tilde{a}_2 x_{i,2} + \cdots + \tilde{a}_n x_{i,n}, i = 1, 2, \ldots, m. \tag{6}$$

where $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)^T$. In this work, the least square regression model is considered to minimize the sum of squared errors of the estimated values based on their specifications [11]. This approach of fuzzy regression is an extension of the classical least squares for minimizing the distance of the observations of the input-output data set. By collecting the above m equations, in matrix format, the following FLS is obtained:

$$\dot{y} = X\tilde{a}, \tag{7}$$

where $X = (X_1, X_2, \ldots, X_m)^T$, $\dot{y} = (\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_m)^T$ and $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)^T$.

Now, by computing $\tilde{a}$ as the solution of (7), the model of fuzzy linear regression of (5) is illustrated. The following theorem explains how a meaningful estimation for the solution of (7) can be computed.

Theorem 1 (7) *Let* X *is a nonnegative crisp matrix.* $\tilde{a}$ *is the solution of* (7) *iff* $\tilde{a}$ *is the solution of*

$$X^T X \tilde{a} = X^T \dot{y}. \tag{8}$$

Due to this theorem, a fuzzy regression analysis is transferred into a fuzzy linear system for computing $\tilde{a}$. So, in the simplest case, a crisp linear system of (4) is solved instead of solving (8) directly.

Remark 2 *There always exist a solution to the problem $X^T X\tilde{a} = X^T\dot{y}$ iff $X^T X$ has full rank.*

To obtain $\tilde{a}$, the linear system $X^T X\tilde{a} = X^T\dot{y}$ has to be solved appropriately. Here, the linear system (8) *is solved by a ST-decomposition technique where $X^T X = ST$ with a symmetric matrix S and a triangular matrix T as described below. Finally, the solution of* (8) *is computed by applying the inverses of S and T on the corresponding crisp linear system with the structure of* (4).

To explain ST decomposition, a 4×4 square matrix A is considered. Let $A = ST$ where

$$A=\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4}\\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4}\\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4}\\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4}\end{pmatrix}, S=\begin{pmatrix} a_{1,1} & a_{2,1} & a_{3,1} & a_{4,1}\\ a_{2,1} & s_{2,2} & s_{2,3} & s_{2,4}\\ a_{3,1} & s_{2,3} & s_{3,3} & s_{3,4}\\ a_{4,1} & s_{2,4} & s_{3,4} & s_{4,4}\end{pmatrix}, T=\begin{pmatrix} 1 & t_{1,2} & t_{1,3} & t_{1,4}\\ 0 & 1 & t_{2,3} & t_{2,4}\\ 0 & 0 & 1 & t_{3,4}\\ 0 & 0 & 0 & 1\end{pmatrix}.$$

Due to the structure of S and T, their elements are obtained as follows.

$$t_{1,2} = (a_{1,2} - a_{2,1})/a_{1,1}, s_{2,2} = (a_{2,2}a_{1,1} - a_{1,2}a_{2,1} + a_{2,1}^2)/a_{1,1},$$

$$s_{2,3} = (a_{3,2}a_{1,1} - a_{1,2}a_{3,1} - a_{3,1}a_{2,1})/a_{1,1}, s_{2,4} = (a_{4,2}a_{1,1} - a_{1,2}a_{4,1} - a_{4,1}a_{2,1})/a_{1,1},$$

$$t_{2,3} = (a_{1,1}(a_{2,3} - s_{2,3}) - a_{2,1}(a_{1,3} - a_{3,1}))/(a_{2,2}a_{1,1} - a_{1,2}a_{2,1}), t_{1,3} = (a_{1,3} - a_{3,1} - a_{2,1}t_{2,3})/a_{1,1},$$

$$s_{3,3} = a_{3,3} - a_{3,1}t_{1,3} - s_{2,3}t_{2,3}, s_{3,4} = a_{4,3} - a_{4,1}t_{1,3} - s_{2,4}t_{2,3},$$

$$t_{1,4} = \frac{(a_{1,4} - a_{4,1})(s_{2,2}s_{3,3} - s_{2,3}^2) + (a_{2,4} - s_{2,4})(a_{3,1}s_{2,3} - a_{2,1}s_{3,3}) + (a_{3,4} - s_{3,4})(a_{2,1}s_{2,3} - a_{3,1}s_{2,2})}{a_{1,1}(s_{2,2}s_{3,3} - s_{2,3}^2) + 2a_{2,1}a_{3,1}s_{2,3} - s_{2,2}a_{3,1}^2 - s_{3,3}a_{2,1}^2},$$

$$t_{2,4} = \frac{(a_{1,4} - a_{4,1})(s_{2,3}a_{3,1} - s_{3,3}a_{2,1}) + (a_{2,4} - s_{2,4})(a_{1,1}s_{3,3} - a_{3,1}^2) + (a_{3,4} - s_{3,4})(a_{3,1}a_{2,1} - a_{1,1}s_{2,3})}{a_{1,1}(s_{2,2}s_{3,3} - s_{2,3}^2) + 2a_{2,1}a_{3,1}s_{2,3} - s_{2,2}a_{3,1}^2 - s_{3,3}a_{2,1}^2},$$

$$t_{3,4} = \frac{(a_{1,4} - a_{4,1})(s_{2,3}a_{2,1} - s_{2,2}a_{3,1}) + (a_{2,4} - s_{2,4})(a_{2,1}a_{3,1} - a_{1,1}s_{2,3}) + (a_{3,4} - s_{3,4})(a_{1,1}s_{2,2} - a_{2,1}^2)}{a_{1,1}(s_{2,2}s_{3,3} - s_{2,3}^2) + 2a_{2,1}a_{3,1}s_{2,3} - s_{2,2}a_{3,1}^2 - s_{3,3}a_{2,1}^2},$$

$$s_{4,4} = a_{4,4} - a_{4,1}t_{1,4} - s_{2,4}t_{2,4} - s_{3,4}t_{3,4}.$$

There is a study about ST decomposition method for fully fuzzy linear system of equations [12]. For FLR, at first the problem $X^T X\tilde{a} = X^T\dot{y}$ is written as BA = Y where the matrix B is generated based on the structure of (4), A $= \begin{pmatrix} \underline{a} \\ -\bar{a}\end{pmatrix}$ and $Y = \begin{pmatrix} \underline{Y} - \bar{Y}\end{pmatrix}$.

Now, the ST-decomposition of matrix B is computed. Finally, the solution of $ST\,B = Y$ has to be solved, by applying the inverses of triangular matrix T and symmetric matrix S on the right hand side crisp vector $Y = \begin{pmatrix} \underline{Y} \\ -\bar{Y}\end{pmatrix}$ where $Y = \begin{pmatrix} \underline{X^T\dot{y}} \\ -\overline{X^T\dot{y}}\end{pmatrix}$, by the following relation

$$A = T^{-1}S^{-1}Y = T^{-1}S^{-1}\begin{pmatrix} \underline{X^T\dot{y}} \\ -\overline{X^T\dot{y}}\end{pmatrix}. \tag{9}$$

4 Numerical Experiments

To test this algorithm, some popular examples are studied. As ST-decomposition is a new algorithm for fuzzy linear system of equations, at first example, the following popular problem is selected to be solved by this method.

Example 1 *The following* 2×2 *FLS is considered.*

$$\begin{aligned} \tilde{x}_1 - \tilde{x}_2 &= (\ r\ \ , 2-r)\,, \\ \tilde{x}_1 + 3\tilde{x}_2 &= (4+r, 7-2r)\,, \end{aligned}$$

Due to the algorithm, in matrix form, we have$A\tilde{x} = \dot{y}$ *and consequently* $BX = Y$ *where*

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}, \tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix}, \dot{y} = \begin{pmatrix} (\ r\ \ , 2-r) \\ (4+r, 7-2r) \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 3 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}, Y = \begin{pmatrix} r \\ 4+r \\ r-2 \\ 2r-7 \end{pmatrix}.$$

Now, the equation $BX = Y$ *is solved by ST-decomposition, i.e.* $X = T^{-1}S^{-1}Y$ *where* $B = ST$ *with the following* S *and* T*.*

$$S = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 1.3333 & 1 \\ 0 & 0 & 1 & 3.6667 \end{pmatrix}, T = \begin{pmatrix} 1 & -1 & 0.3333 & 1.1111 \\ 0 & 1 & -0.3333 & -0.1111 \\ 0 & 0 & 1 & -0.6667 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Therefore,

$$X = T^{-1}S^{-1}Y = \begin{pmatrix} 0.625r + 1.375 \\ 0.125r + 0.875 \\ 0.875r - 2.875 \\ 0.375r - 1.375 \end{pmatrix}.$$

The following example is selected from [13] for fuzzy linear regression.

Example 2 *The following geology data set is from a study about the effect of Sodium absorption rate (x: SAR as crisp input variable) on percent of Sodium exchange (y: PSE as triangular fuzzy output) in Salikhor region of Lorestan province of Iran in 2001 [13]. Due to unconcluded with sufficient accuracy in measurements of PSE, the observations related to yare ambiguous. The observation data set of SAR and their corresponding PSE values are represented in the above table. So, a FLR is an appropriate model for the relation of PSE parameters based on SAR information as*

$$\tilde{y} = \tilde{a}_0 + \tilde{a}_1 x_1, \tag{10}$$

with unknown fuzzy parameters $\tilde{a}_0$ *and* $\tilde{a}_1$*. From the* (6), (7) *and* (8), *in matrix form, we have* $X_i = (1, x_i)^T$ *and data set* $(X_i, \tilde{y}_i)$*where* $X_i = (1, x_1, x_2, \ldots, x_n)^T$ *so that* $X_i^T \tilde{a} = \tilde{y}_i$ *where* $\tilde{a} = (\tilde{a}_0, \tilde{a}_1)^T$ *i.e.* $\tilde{y}_i = \tilde{a}_0 + \tilde{a}_1 x_i$, $i = 1, 2, \ldots, 24$. *So, for* $X^T X \tilde{a} = X^T \dot{y}$ *we have*

$$\begin{pmatrix} 24 & 20.91 \\ 20.91 & 26.8601 \end{pmatrix} \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix} = \begin{pmatrix} (164.09 - 119.98r, 94.21 + 119.98r) \\ (202.3708 - 165.2557r, 88.364 + 165.2557r) \end{pmatrix},$$

The above fuzzy linear system is transferred into a 4×4*crisp linear system* (4). *By applying the ST-decomposition, i.e.* $B = ST$ *, the regression parameters of* (10) *are computed from* (9) *as*

$$\begin{aligned} \tilde{a}_0 &= (0.8202 + 32.3237\,\mathrm{r}\,,\ 21.1883 + 32.3237\,\mathrm{r})\,, \\ \tilde{a}_1 &= (6.9060 - 31.3625\,\mathrm{r}\,,\ -19.8139\ - 31.3625\ \mathrm{r})\,. \end{aligned}$$

Table 1: Data information of example 2 (for crisp inputs x_i and triangular fuzzy outputs $\tilde{y}_i$.

No.(i)	SAR:x_i	EPS: $\tilde{y}_i = (y_i, s_i, s'_i)$	No.(i)	SAR: x_i	EPS: $\tilde{y}_i = (y_i, s_i, s'_i)$
1	0.78	$(3.08, 0.31, 0.62)$	13	0.71	$(5.23, 0.52, 1.05)$
2	0.64	$(2.86, 0.29, 0.57)$	14	0.51	$(5.16, 0.52, 1.03)$
3	0.62	$(6.25, 0.62, 1.25)$	15	0.77	$(11.10, 1.11, 2.22)$
4	0.49	$(4.11, 0.41, 0.82)$	16	0.99	$(4.74, 0.47, 0.95)$
5	1.10	$(1.04, 0.10, 0.21)$	17	3.56	$(28.84, 2.88, 5.77)$
6	0.61	$(2.71, 0.28, 0.54)$	18	0.86	$(9.43, 0.94, 1.89)$
7	0.74	$(4.45, 0.45, 0.89)$	19	0.61	$(4.50, 0.45, 0.90)$
8	1.15	$(6.92, 0.69, 1.38)$	20	0.64	$(9.30, 0.92, 1.86)$
9	1.08	$(7.41, 0.74, 1.48)$	21	0.71	$(9.48, 0.95, 1.90)$
10	0.38	$(9.08, 0.91, 1.82)$	22	0.61	$(3.65, 0.36, 0.73)$
11	0.61	$(6.56, 0.66, 1.31)$	23	0.63	$(10.14, 1.01, 2.03)$
12	0.98	$(5.05, 0.50, 1.01)$	24	1.13	$(3.00, 0.30, 0.60)$

5 Conclusion

Fuzzy linear regression based on the least squares based on the least squares of errors is a popular fuzzy regression model. In this work, we studied ST-decomposition method for solving fuzzy linear system of equations and later on, we apply this method for estimating the fuzzy linear regression parameters. This method is a simple and strong method that our numerical examples confirm the applicability of this method for such a FLR problems.

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