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A Nonlinear EOQ Model with the Effect of Trade Credit

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Abstract: An economic order quantity model has been developed for a constantly deteriorating item for which the supplier permits a fixed delay in payments or in other words trade credit, the demand rate being time-dependent. Different decision making situations are illustrated with the help of numerical examples. Sensitivity analysis of the optimal solution with respect to changes in the parameter values is carried out.

Keywords: Inventory; Economic order quantity; Permissible delay in payments; Deterioration; Time-dependent demand rate.

1 Introduction

In today's dynamic environment, in most cases the demand for items increases with time. Many companies are working towards increasing demand of their items with time. So such a demand rate is considered in this model. It is again normally considered that the items stored for future use, always lose part of their value constantly with time, which is known as deterioration in the language of inventory. In real life, many items such as milk, blood, drugs, vegetables, fruits, foodstuffs etc. undergo direct spoilage while kept in store. With the passage of time, electronic goods, radioactive substances, photographic film, grain etc. deteriorate through gradual loss of utility or potential with the passage of time. Also highly volatile liquids like alcohol, turpentine, gasoline etc. undergo depletion with time due to evaporation. So we see that decay or deterioration of physical goods in stock is a very realistic feature. This paper has incorporated both the features of increasing demand and constant deterioration with time in the model. There has been many mathematical developments in the past 30 years for a finite time horizon inventory model for deteriorating items with time varying demand. Many important earlier works in this field will be found in the references Donaldson [1], Dave and Patel [2], Sachan [3], Goswami and Chaudhuri [4], Goyal et al [5], Wee [6], Benkherouf [7], Hariga [8], Giri et al. [9] and Chung and Tsai [10]. All these studies however assumed the payment for the goods will be made to the vendor immediately after receiving the consignment, which is not quite practical in the real world. In most marketing situations, a vendor often provides buyers with a trade credit period to stimulate the demand, boost market share or decrease inventories of certain items.

The problem of determining the economic order quantity under the condition of a permissible delay in payment has drawn the attention of researchers in recent times. It is assumed that the supplier (whole-saler) allows a delay of a fixed period, that is the trade credit period, for settling the amount owed to him. There is no interest charged on the outstanding amount if it is paid within the permissible delay period. Beyond this period, interest is charged. During this fixed period of permissible delay in payments, the customer (a retailer) can sell the items, invest the revenues in an interest-earning account and earn interest instead of paying off the over-draft which is necessary if the supplier requires settlement of the account immediately after replenishment. The customer finds it economically beneficial to delay the settlement to the last moment of the permissible period of delay. The effect of supplier credit policies on optimal order quantity has received the attention of many researchers such as Aggarwal and Jaggi [11], Chang and Dye [12], Chang et al.[13], Chen and Chuang [14], Chu et al. [15], Chung [16, 17, 18], Goyal [19], Jamal et al.[20, 21], Khouja and Mehrez [22], Liao et al.[23], Sarkar et al.[24, 25] and Shah and Shah [26]. This problem was first studied by Goyal [19] for a non-deteriorating item having a constant demand rate. Chand and Ward [27] communicated, in a brief note, on some of the assumptions made by Goyal [19] in analyzing the cost of funds tied up in inventory. The effects of deterioration of goods in stock on the cost and price components cannot be ignored in practice. The model of Goyal [19] was extended by

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Aggarwal and Jaggi [11] to the case of a deteriorating item. Basu and Sinha [28] and more recently Tripathy and Misra [29] and Misra et. al.[30] derived optimal replenishment policies for two parameters Weibull deteriorating items with the permissible delay in payment. Hwang and Shinn [31] discussed lot sizing policy for an exponentially deteriorating product under the condition of permissible delay in payments when the demand rate would depend on retail price. The optimal inventory policies under trade credits depending on the ordering quantity was investigated by Khouja and Mehrez [22] and extended by Chung, Goyal and Huang [32]. Inventory models for exponentially increasing demand with time [33] or in demand declining market [34] and Weibull demand [35] with effect of delay in payments have also been studied by some researchers. Several studies have also examined the inflationary effect on an inventory policy. Liao et. al.[23] developed an inventory model with deteriorating items under inflation when a delay in payment is permissible. Again Ouyang et. al.[36] as well as Tripathi & Kumar [37] considered cash discounts for items under trade credits, which may be allowed by certain suppliers.

In the present paper, the model of Aggarwal and Jaggi [11] is reconsidered and extended taking into account a time-dependent demand rate . The solution procedure involving different decision making situations is illustrated with the help of numerical examples. Analysis is carried out to study the sensitivity of the solution to changes in the values of the different parameters involved in the system.

2 Notations and assumptions in the model

The following notations are used in this model:

D(t) = demand rate which is linearly dependent on time i.e. D(t) = a + bt where a ¿ 0, b ¿ 0

h_p= holding cost (excluding interest charges) per rupee of unit purchase cost per unit time

p = unit purchase cost of an item in rupees

s = cost of placing an order in rupees (set-up cost)

I_p= interest charges per rupee investment in stocks per year

I_e= interest earned per rupee in a year

t_1 = permissible period (in years) of delay in settling the accounts with the supplier

T = time interval (in years) between two successive orders

θ= constant rate of deterioration of an item.

The following assumptions are made in the development of the model:

(i) The demand rate for the item is represented by a linear and continuous function of time.

(ii) Shortages are not allowed.

(iii) Depletion of inventory over time due to both demand and deterioration occurs simultaneously.

(iv) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. At the end of this period, the account is settled and the customer starts paying for the interest charges on the items in stock.

(v) Time horizon is infinite.

(vi) The inventory system involves only one type of item.

(vii) Replenishments are instantaneous with a known and constant lead time.

The total annual variable cost consists of the following elements:

(1) Cost of placing orders.

(2) Cost of stock holding (excluding interest charges) per year.

(3) The interest earned during the permissible settlement period for the two cases when T $\geq t_1$ and T ¡t_1.

(4) Cost of interest charges for the items kept in stock. As items are sold before settlement of the replenishment account, the sales revenue is used to earn interest. When the replenishment account is settled, the situation is reversed, and effectively the items still in stock have to be financed at interest rate I_p. The interest is payable during the time (T – t_1).

(5) The cost of deteriorated units.

3 The Model

We first calculate the stock level $q(t)$ at any time t when the stock deteriorates at a constant rate θ, 0 ¡ θ ¡ 1. The instantaneous state of the stock level $q(t)$ is governed by the differential equation

$$\frac{dq(t)}{dt} + \theta q(t) = -D(t), 0 \leq t \leq T \tag{1}$$

where

$$q(0) = q_0, q(T) = 0 \tag{2}$$

and

$$D(t) = a + bt,\ where\ a > 0, b > 0. \tag{3}$$

The solution of (1) using$(2)$and (3) is

$$q(t) = \frac{1}{\theta}[e^{\theta(T-t)}(a - \frac{b}{\theta} + bT) - (a - \frac{b}{\theta} + bt)]. \tag{4}$$

Putting $a = \lambda$, $b = 0$, this equation corresponds to the equation (4) in Aggarwal and Jaggi [1].
The initial order quantity at $t = 0$ is

$$q_0 = q(0) = \frac{1}{\theta}[e^{\theta T}(a - \frac{b}{\theta} + bT) - (a - \frac{b}{\theta})]. \tag{5}$$

The total demand during one cycle is $\int_0^T (a + bt)dt = aT + \frac{bT^2}{2}$.
Number of deteriorated units is equal to $q_0 - (aT + \frac{bT^2}{2})$

$$= \frac{1}{\theta}[e^{\theta T}(a - \frac{b}{\theta} + bT) - (a - \frac{b}{\theta})] - \frac{T}{2}(2a + bT). \tag{6}$$

The cost of stock holding for one cycle = $h \int_0^T q(t)dt$, where $h = ph_p$.
$= \frac{h}{\theta} \int_0^T [e^{\theta(T-t)}(a - \frac{b}{\theta} + bT) - (a - \frac{b}{\theta} + bt)]dt$
$= \frac{h}{\theta}[\frac{(a - \frac{b}{\theta} + bT)}{\theta}(e^{\theta T} - 1) - T(a - \frac{b}{\theta} + \frac{bT}{2})]$.
Hence, the holding cost per unit time is

$$\frac{h}{T\theta}[\frac{(a - \frac{b}{\theta} + bT)}{\theta}(e^{\theta T} - 1) - T(a - \frac{b}{\theta} + \frac{bT}{2})] \tag{7}$$

where$h = ph_p$.

$\mathbf{Case - I}$: Let$T > t_1$.

Since the interest is payable during the time $(T - t_1)$, the interest payable in one cycle is

$$pI_p \int_{t_1}^{T} q(t)dt = \frac{pI_p}{\theta} \int_{t_1}^{T} [e^{\theta(T-t)}(a - \frac{b}{\theta} + bT) - (a - \frac{b}{\theta} + bt)]dt$$

$$= \frac{pI_p}{\theta}[\frac{1}{\theta}(a - \frac{b}{\theta} + bT)\left\{e^{\theta(T-t_1)} - 1\right\} - (T - t_1)\left\{a - \frac{b}{\theta} + \frac{b}{2}(T + t_1)\right\}].$$

Hence, interest payable per unit time is

$$\frac{pI_p}{\theta T}[\frac{1}{\theta}(a - \frac{b}{\theta} + bT)\left\{e^{\theta(T-t_1)} - 1\right\} - (T - t_1)\left\{a - \frac{b}{\theta} + \frac{b}{2}(T + t_1)\right\}]. \tag{8}$$

Interest earned per unit time is

$$\frac{pI_e}{T}\int_0^T tD(t)dt = pI_eT(\frac{a}{2} + \frac{bT^2}{3}). \tag{9}$$

Total variable cost per cycle = Ordering cost +cost of deteriorated units + inventory holding cost + interest payable beyond the permissible period - interest earned during the cycle.

Hence, the $total\, variable\, cost\, per\, unit\, time$ in this case is given by

$$z_1(T) = \frac{s}{T} + \frac{p}{T}[\frac{1}{\theta}\{e^{\theta T}(a - \frac{b}{\theta} + bT) - (a - \frac{b}{\theta})\} - \frac{T}{2}(2a + bT)]$$
$$+\frac{h}{T\theta}[\frac{1}{\theta}(a - \frac{b}{\theta} + bT)(e^{\theta T} - 1) - T(a - \frac{b}{\theta} + \frac{bT}{2})]$$
$$+\frac{pI_p}{T\theta}[\frac{1}{\theta}(a - \frac{b}{\theta} + bT)\{e^{\theta(T-t_1)} - 1\} - \{(a - \frac{b}{\theta})(T - t_1)$$
$$+\frac{b}{2}(T^2 - t_1^2)\}] - pI_eT(\frac{a}{2} + \frac{bT}{3}). \tag{10}$$

We have now to minimize $z_1(T)$ for a given value of t_1.

The necessary and sufficient conditions to minimize $z_1(T)$ for a given value of t_1 are respectively

$$\frac{dz_1(T)}{dT} = 0 \tag{11}$$

and

$$\frac{d^2z_1(T)}{dT^2} > 0. \tag{12}$$

After simplification, $\frac{dz_1(T)}{dT} = 0$ yields the following non-linear equation in T:

$$2b\theta[p\theta + h + pI_pe^{-\theta t_1}]T^2e^{\theta T} + 2\theta[p\theta(a - \frac{b}{\theta}) + h(a - \frac{b}{\theta}) + pI_p(a - \frac{b}{\theta})e^{-\theta t_1}]Te^{\theta T}$$
$$-2(a - \frac{b}{\theta})[p\theta + h + pI_pe^{-\theta t_1}]e^{\theta T} - \frac{4}{3}pI_eb\theta^2T^3 - \theta[b(h + p\theta) + paI_e\theta + pbI_p]T^2$$
$$+2(a - \frac{b}{\theta})[h + p\theta - pI_p\theta t_1 + pI_p] - 2s\theta^2 - pbI_p\theta t_1^2 = 0. \tag{13}$$

By solving (13) for T, using Newton–Raphson method we obtain the optimal cycle length T_1^*, provided it satisfies (12).

The EOQ q_0^* for this case is given by

$q_0^*(T_1^*) = q(0) = \frac{1}{\theta}[e^{\theta T_1^*}(a - \frac{b}{\theta} + bT_1^*) - (a - \frac{b}{\theta})]$.

The minimum annual variable cost $z_1(T_1^*)$ is then obtained from (10) for $T = T_1^*$.

Case – II : $T < t_1$.

In this case, the customer earns interest on the sales revenue up to the permissible delay period and no interest is payable during this period for the items kept in stock.

Interest earned up to T is

$$pI_e\int_0^T tD(t)dt = pI_eT^2(\frac{a}{2} + \frac{bT^3}{3})$$

and interest earned during $(t_1 - T)$ i.e. up to the permissible delay period is

$$pI_e(t_1 - T)\int_0^T D(t)dt = pI_e(a + \frac{bT}{2})(t_1 - T)T.$$

Hence the total interest earned during the cycle is

$$pI_eT^2(\frac{a}{2} + \frac{bT}{3}) + pI_e(a + \frac{bT}{2})(t_1 - T)T$$
$$= pI_eT\{\frac{1}{2}(bt_1 - a)T - \frac{1}{6}bT^2 + at_1\}. \tag{14}$$

Total variable cost per cycle = Ordering cost + cost of deteriorated units + inventory holding cost – interest earned during the cycle.

Hence, the total variable cost per unit time in this case is

$$z_2(T)=\frac{s}{T}+\frac{p}{T}[\frac{1}{\theta}\{e^{\theta T}(a-\frac{b}{\theta}+bT)-(a-\frac{b}{\theta})\}-\frac{T}{2}(2a+bT)]$$

$$+\frac{h}{T\theta}[\frac{(a-\frac{b}{\theta}+bT)}{\theta}(e^{\theta T}-1)-T(a-\frac{b}{\theta}+\frac{bT}{2})]$$

$$-pI_e[(bt_1-a)\frac{T}{2}-\frac{1}{6}bT^2+at_1]. \tag{15}$$

We have now to minimize $z_2(T)$ as before for a given value of t_1.

After some simplification, $\frac{dz_2(T)}{dT}=0$ yields the result

$$-s\theta-p\theta[\frac{1}{\theta}\{e^{\theta T}(a-\frac{b}{\theta}+bT)-(a-\frac{b}{\theta})\}-\frac{T}{2}(2a+bT)]+p\theta T[\frac{1}{\theta}\{be^{\theta T}+(a-\frac{b}{\theta}+bT)\theta e^{\theta T}\}-(a+bT)]$$

$$-h[\frac{1}{\theta}(a-\frac{b}{\theta}+bT)(e^{\theta T}-1)-T(a-\frac{b}{\theta}+\frac{bT}{2})]+hT[(a-\frac{b}{\theta}+bT)e^{\theta T}+\frac{1}{\theta}be^{\theta T}-(a+bT)]$$

$$-pI_e\theta T^2[\frac{1}{2}(bt_1-a)-\frac{1}{3}bT]=0. \tag{16}$$

The optimal cycle length $T=T_2^*$ which minimizes$z_2(T)$ is obtained by solving the equation (16) for T using Newton-Raphson method, provided $\frac{d^2z_2(T)}{dT^2}>0$.

The EOQ in this case is given by

$$q_0^*(T_2^*)=\frac{1}{\theta}[e^{\theta T_2^*}(a-\frac{b}{\theta}+bT_2^*)-(a-\frac{b}{\theta})]$$

and the minimum annual variable cost $z_2(T_2^*)$ is obtained from (15) for $T=T_2^*$.

Case – III : $T=t_1$.

For $T=t_1$, both the cost functions $z_1(T)$ and $z_2(T)$ become identical and the cost function is then denoted by $z(t_1)$, say. $z(t_1)$ is obtained on substituting $T=t_1$ either in (10) or in (15).

Thus

$$z(t_1)=\frac{s}{t_1}+\frac{p}{t_1}[\frac{1}{\theta}\{e^{\theta t_1}(a-\frac{b}{\theta}+bt_1)-(a-\frac{b}{\theta})\}-\frac{t_1}{2}(2a+bt_1)]$$

$$+\frac{h}{\theta t_1}[\frac{1}{\theta}(a-\frac{b}{\theta}+bt_1)(e^{\theta t_1}-1)-t_1(a-\frac{b}{\theta}+\frac{bt_1}{2})]-\frac{1}{6}pI_et_1(3a+2bt_1). \tag{17}$$

The EOQ is

$$q_0^*(t_1)=\frac{1}{\theta}[e^{\theta t_1}(a-\frac{b}{\theta}+bt_1)-(a-\frac{b}{\theta})]$$

.

Now in order to obtain the economic ordering policy, the following steps are to be followed:

Step 1: Determine $T_1^*$from (13). If $T_1^*\geq t_1$, obtain $z_1(T_1^*)$ from (10).

Step 2: Determine $T_2^*$from (16). If $T_2^*<t_1$, obtain $z_2(T_2^*)$ from (15).

Step 3: If $T_1^*<t_1$and $T_2^*\geq t_1$, then evaluate $z(t_1)$ from (17).

Step 4: Compare $z_1(T_1^*)$, $z_2(T_2^*)$ and $z(t_1)$ and take the minimum.

4 Numerical illustrations

The solution procedure involving different decision making situations are illustrated by the examples, covering all the three cases that arise in the model:

Example 1 *Let* $a = 1000$ *units per year,*$b = 150$ *units per year,* $I_p = 0.15$ *per year,*$I_e = 0.13$ *per year,*$s = Rs.200$ *per order,*$h_p = Rs.0.12$ *per year,* $p = Rs.20$ *per unit,*$t_1 = 0.25$*year,* $\theta = 0.20$.

Solving (13) *for T, we obtain the optimal value* $T_1^* = 0.284$ *and the minimum average cost is* $z_1(T_1^*) = 1283.53$. *Again solving* (16)*, we have* $T_2^* = 0.206$ *and the minimum average cost is* $z_2(T_2^*) = 1263.53$. *Here* $T_1^* > t_1$ *and* $T_2^* < t_1$ *both hold and this implies that both the example 1 hold. Now* $z_2(T_2^*) < z_1(T_1^*)$. *Hence the minimum average cost in this case is* $z_2(T_2^*) = Rs.1263.53$, *where the optimal cycle length is* $T_2^* = 0.206\,year < t_1$. *The EOQ is given by* $q_0^*(T_2^*) = 213.82\,units$.

Example 2 *Let*$a = 1000$ *units per year,*$b = 150$ *units per year,* $I_p = 0.15$ *per year,*$I_e = 0.13$ *per year,* $s = Rs.200$ *per order,*$h_p = Rs.0.12$ *per year,* $p = Rs.20$ *per unit,* $t_1 = 0.25$*year,*$\theta = 0.01$.

Solving(13)*for T, we get* $T_1^* = 0.432$ *and the minimum average cost is* $z_1(T_1^*) = 585.31$. *Again solving* (16)*, we have* $T_2^* = 0.274$ *and the corresponding minimum average cost is* $z_2(T_2^*) = 793.94$. *Here* $T_2^* > t_1$ *which contradicts example 2. In this case* $T_1^* > t_1$ *which is case*I*. Therefore, the minimum average cost in this case is* $z_1(T_1^*) = Rs.585.31$*, the EOQ is* $q_0^*(T_1^*)$*=* 447.23 *units and the optimal cycle length is* $T_1^* = 0.432$*, year¿*t_1.

Example 3 *Let*$a = 1000$ *units per year,*$b = 150$ *units per year,* $I_p = 0.154$ *per year,*$I_e = 0.13$ *per year,*$s = Rs.200$ *per order ,*$h_p = Rs.0.12$ *per year,* $p = Rs.40$ *per unit,*$t_1 = 0.25$ *year,*$\theta = 0.20$.

Solving (13) *for T, we obtain the optimal value* $T_1^* = 0.232$ *and the optimal cost* $z_1(T_1^*)$*=* 1792.29. *Here* $T_1^* < t_1$ *which contradicts case* I*. Again solving* (16)*, we have* $T_2^* = 0.147$*; and the minimum average cost is* $z_2(T_2^*) = 1395.29$. *In this case,* $T_2^* < t_1$ *which is example 2. Therefore, the minimum average cost in this case is* $z_2(T_2^*) = 1395.29$*, the EOQ is* $q_0^*(T_2^*) = 150.81$ *and the optimal cycle length is* $T_2^* = 0.147 < t_1$.

Example 4 *Let*$a = 1300$ *units per year,*$b = 100$ *units per year,* $I_p = 0.5$ *per year,*$I_e = 0.01$ *per year,*$s = Rs.97$*per order,*$h_p = Rs.0.12$*, per year,* $p = Rs.40$ *per unit,*$t_1 = 0.09$ *year,*$\theta = 0.3$.

In this case, $T_1^* = T_2^* = 0.09 = t_1$*,which is example 3.*

The optimal cost in this case is $z(t_1) = Rs.2050.56$ *and the EOQ is* $q_0^* = 119.01$ *units for* $t_1 = .09\,year$.

5 Sensitivity analysis

A sensitivity analysis is carried out using the parameter values of $Example - III$ and the results are shown in Table-I. It is found that the solution is not sensitive to changes in the values of the parameters b and I_p. However, it is sensitive to changes in the values of the parameters a, h_p, I_e, p, s, θ and t_1.

Again taking the parameter values $a = 1000\,units\,per\,year$, $b = 150\,units\,per\,year$, $I_p = 0.15\,per\,annum$, $I_e = 0.13\,per\,annum$, $s = Rs.200\,per\,order$, $h_p = Rs.0.12\,per\,year$ (as in the numerical $Example - III$), we present the optimal solutions on varying t_1 and θ for a $less\,expensive$ item ($p = Rs.20\,per\,unit$) [shown in Table-II] to a $more\,expensive$ item ($p = Rs.40\,per\,unit$) [shown in Table-III].

We observe the following similar type of characteristics of the solution in both the cases:

(i) The cycle length increases marginally, the order quantity increases slightly and the cost decreases slightly as the credit period t_1 increases, keeping θ fixed.

(ii) As the value of θ increases, keeping the credit period t_1 fixed, there is significant reduction in both the cycle length and the order quantity while the cost increases considerably.

However for a $very\ expensive$ item ($p = Rs.200/unit$) [shown in Table-IV] it is seen that the cycle length, the order quantity and the average system cost all undergo considerable changes.

6 Conclusions

This model shows that there are slight changes in the optimal solutions when a time-dependent demand rate is taken into consideration rather than a constant demand rate.As a result of this demand rate both cycle length and order quantity

Table 1: Sensitivity of the optimal solution to changes in parameter values.

$Param-eters$	$\%change\ inpara-meters$	T_1^*	$z_1(T_1^*)$	T_2^*	$z_2(T_2^*)$	$Remarks$	$Solution$
a	50	0.213	2229.626	0.121	1345.365	$T_2^* < t_1$	$z_2(T_2^*)$
	20	0.223	1971.013	0.135	1389.847	$T_2^* < t_1$	$z_2(T_2^*)$
	-20	0.246	1605.640	0.164	1374.495	$T_2^* < t_1$	$z_2(T_2^*)$
	-50	0.279	1299.320	0.204	1269.928	$T_1^* > t_1, T_2^* < t_1$	$z_2(T_2^*)$
b	50	0.231	1802.880	0.146	1396.504	$T_2^* < t_1$	$z_2(T_2^*)$
	20	0.232	1796.539	0.147	1395.781	$T_2^* < t_1$	$z_2(T_2^*)$
	-20	0.233	1788.024	0.147	1394.798	$T_2^* < t_1$	$z_2(T_2^*)$
	-50	0.234	1781.591	0.147	1394.047	$T_2^* < t_1$	$z_2(T_2^*)$
h_p	50	0.215	2070.220	0.138	1570.236	$T_2^* < t_1$	$z_2(T_2^*)$
	20	0.225	1906.244	0.143	1466.622	$T_2^* < t_1$	$z_2(T_2^*)$
	-20	0.241	1674.154	0.151	1321.984	$T_2^* < t_1$	$z_2(T_2^*)$
	-50	0.256	1488.002	0.158	1207.922	$T_1^* > t_1, T_2^* < t_1$	$z_2(T_2^*)$
I_e	50	0.257	1467.189	0.138	924.703	$T_1^* > t_1, T_2^* < t_1$	$z_2(T_2^*)$
	20	0.241	1666.209	0.143	1208.435	$T_2^* < t_1$	$z_2(T_2^*)$
	-20	0.224	1913.756	0.151	1580.138	$T_2^* < t_1$	$z_2(T_2^*)$
	-50	0.214	2088.299	0.158	1853.247	$T_2^* < t_1$	$z_2(T_2^*)$
I_p	50	0.236	1793.970	0.147	1395.292	$T_2^* < t_1$	$z_2(T_2^*)$
	20	0.234	1793.044	0.147	1395.292	$T_2^* < t_1$	$z_2(T_2^*)$
	-20	0.231	1791.388	0.147	1395.292	$T_2^* < t_1$	$z_2(T_2^*)$
	-50	0.228	1789.846	0.147	1395.292	$T_2^* < t_1$	$z_2(T_2^*)$
p	50	0.212	2238.779	0.120	1345.124	$T_2^* < t_1$	$z_2(T_2^*)$
	20	0.223	1974.962	0.134	1390.033	$T_2^* < t_1$	$z_2(T_2^*)$
	-20	0.246	1600.895	0.164	1373.515	$T_2^* < t_1$	$z_2(T_2^*)$
	-50	0.284	1283.529	0.206	1263.527	$T_1^* > t_1,\ T_2^* < t_1$	$z_2(T_2^*)$
s	50	0.259	2198.801	0.179	2008.266	$T_1^* > t_1,\ T_2^* < t_1$	$z_2(T_2^*)$
	20	0.244	1960.307	0.161	1655.291	$T_2^* < t_1$	$z_2(T_2^*)$
	-20	0.221	1615.720	0.132	1108.257	$T_2^* < t_1$	$z_2(T_2^*)$
	-50	0.201	1331.351	0.105	600.396	$T_2^* < t_1$	$z_2(T_2^*)$
θ	50	0.203	2256.024	0.133	1686.162	$T_2^* < t_1$	$z_2(T_2^*)$
	20	0.219	1984.961	0.141	1515.115	$T_2^* < t_1$	$z_2(T_2^*)$
	-20	0.248	1588.024	0.154	1270.099	$T_2^* < t_1$	$z_2(T_2^*)$
	-50	0.278	1254.317	0.168	1070.399	$T_1^* > t_1, T_2^* < t_1$	$z_2(T_2^*)$
t_1	50	0.293	1938.772	0.147	738.118	$T_1^* > t_1, T_2^* < t_1$	$z_2(T_2^*)$
	20	0.256	1831.631	0.147	1132.425	$T_1^* > t_1, T_2^* < t_1$	$z_2(T_2^*)$
	-20	0.211	1788.009	0.147	1658.157	$T_2^* < t_1$	$z_2(T_2^*)$
	-50	0.186	1872.180	0.147	2052.448	$T_2^* < t_1$	$z_2(T_2^*)$

Table 2: Results for p=Rs.20 per unit

$t_1 yrs.$		$\theta = .01$		
	$T_1^* or\, T_2^* (yrs.)$	$z_1(T_1^*)\, or\, z_2(T_2^*)\, (yrs.)$	$q_0^* units$	Optimal cycle length
0	0.352	1115.97	362.11	T_1^*
0.05	0.357	972.35	366.84	T_1^*
0.10	0.367	849.36	378.21	T_1^*
		$\theta = 0.10$		
0	0.277	1415.06	286.81	T_1^*
0.05	0.281	1273.61	290.54	T_1^*
0.10	0.289	1158.93	299.53	T_1^*
		$\theta = 0.20$		
0	0.232	1686.68	241.77	T_1^*
0.05	0.235	1547.23	244.92	T_1^*
0.10	0.242	1440.23	252.49	T_1^*

Table 3: Results for,p=Rs.40 per unit

$t_1 yrs.$		$\theta = 0.01$		
	$T_1^* or\, T_2^* (yrs.)$	$z_1(T_1^*)\, or\, z_2(T_2^*)\, (yrs.)$	$q_0^* units$	Optimal cycle length
0	0.252	1569.92	256.62	T_1^*
0.05	0.257	1293.44	260.36	T_1^*
0.10	0.271	1072.94	277.27	T_1^*
		$\theta = 0.10$		
0	0.198	1989.25	203.15	T_1^*
0.05	0.207	1719.89	207.68	T_1^*
0.10	0.214	1522.66	219.44	T_1^*
		$\theta = 0.20$		
0	0.166	2370.09	171.08	T_1^*
0.05	0.170	2107.29	174.90	T_1^*
0.10	0.180	1931.33	184.79	T_1^*

Table 4: Results for p=Rs.200 per unit

$t_1 yrs.$		$\theta = 0.01$		
	$T_1^* or\, T_2^* (yrs.)$	$z_1(T_1^*)\, or\, z_2(T_2^*)\, (yrs.)$	$q_0^* units$	Optimal cycle length
0	0.114	3485.11	115.14	T_1^*
0.05	0.125	2285.97	125.95	T_1^*
0.10	0.152	1589.08	153.52	T_1^*
		$\theta = 0.10$		
0	0.090	4411.78	91.08	T_1^*
0.05	0.098	3296.12	99.62	T_1^*
0.10	0.075	2697.22	75.93	T_2^*
		$\theta = 0.20$		
0	0.076	5253.46	76.60	T_1^*
0.05	0.083	4213.92	83.78	T_1^*
0.10	0.066	3413.56	67.00	T_2^*

decreases ,whereas the average system cost increases in this model.It is also seen that when the unit purchase cost is high and decay is continuous, the savings due to delayed payment appears to be more significant than when decay is continuous but without delayed payment.

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