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A Solution to the Telegraph Equation by Using DGJ Method

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Abstract: This article explores the utility of the Daftardar-Gejji-Jafaris (DGJ) method to obtain approximate solution of the hyperbolic telegraph equation. The method delivers reliable results in the form of analytical approximation. Thus the method is seen to be a very reliable alternative tool for finding analytical solutions of more intricate equations. Use of small parameter or linearization is not needed in the present approach as opposed to numerical methods. The main goal of this work is to derive approximations to the solution of the telegraph equation. To illustrate the method and its ability six examples are presented. The produced results convergence to the analytical solution and are in very good agreement with the literature. The proposed method is realized to be more cost-effective in terms of computation. The DGJ method is seen to be a very good alternative to existing iterative techniques.

Keywords: Hyperbolic telegraph equation; PDE, new iterative method; Iterative series methods.

1 Introduction

Mathematical modeling of most physical systems leads to linear/nonlinear partial differential equations (PDEs) in various fields of science. A class of the most important linear partial differential equations is the hyperbolic partial differential equations describing the vibrations of structures and being the basis for fundamental equations of atomic physics. Recently, much effort has been spent to develop and implement stable methods for the numerical solution of second-order hyperbolic equations, for instance see [14]. The second-order telegraph equation with constant coefficients, represents mixture between diffusion and wave propagation by introducing a term that explains effects of finite velocity to mass transport equation [5]. However, the equation mostly models signal analysis for transmission and propagation of electrical signals [6]. Recently, the telegraph equation is found to be more suitable than ordinary diffusion equation in modeling the reaction diffusion for such fields of science [3]. To deal with the equation, various mathematical methods have been proposed for obtaining exact and approximate analytic solutions. For instance, Dehghan and Shokri [2] proposed a numerical scheme to solve the one-dimensional hyperbolic telegraph equation using collocation points and approximating the solution using thin plate splines radial basis function. Mohebbi and Dehghan [3] combined a high-order compact finite difference scheme to approximate the spatial derivative and the collocation technique for the time component to numerically solve the one-dimensional linear hyperbolic equation. In the work of Gao and Chi [1], the authors developed a numerical algorithm for the solution of the nonlinear telegraph equations. Numerical solution of variable coefficient telegraph equation was discussed in [7]. Yet, Mohanty and his coworkers [8,9] developed new three-level implicit unconditionally stable alternating direction implicit schemes for the two and three-space-dimensional linear hyperbolic equations. In solving the second-order linear hyperbolic equation, Dehgan and Lakestani [10] used a numerical technique consisting of expanding the approximate solution as the elements of Chebyshev cardinal functions. Biazar et al. [11] applied the variational iteration method to obtain an approximate solution of the telegraph equation. Saadatmandi and Dehghan [12] used the Chebyshev Tau method in numerically solving the telegraph equation. Solutions of the telegraph equation are still an attractive and interesting topic. Therefore the Daftardar-Gejji-Jafaris (DGJ) method [13,14] has been used to produce analytical approximate solutions in the current study. The present method does not encounter the well-known drawbacks of the numerical methods such as linearization, discretization, and selection of small parameter etc. Therefore this work is devoted to find approximate solution of the telegraph equation. The DGJ method is a new approximate method for solving linear/nonlinear partial differential equations without linearization or small perturbations. An analytical solution can be

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obtained from its trial-function successively. In this study, the DGJ method was implemented for obtaining approximate solution of the telegraph equation. All results converge to exact solutions and are in full agreement with the literature. Comparisons show that there is a very good agreement between the present solutions and the exact solutions in terms of accuracy in the last two examples. The present method is seen to be a very good alternative to existing iterative techniques. To the best knowledge of the authors, this method has not been implemented for the problems represented by the telegraph equation so far. The main purpose of this paper is to illustrate the advantages and the simplicity of using the DGJ method over the other iterative methods in terms of comparisons. The structure of this article is as follows. In section 2, the model equation is given while the DGJ method is described briefly in section 3. The convergence of the method is mentioned in section 4. In section 5, the proposed method is used to approximate the solution of the problem and the results of illustrative examples are presented. The last concluding section summarizes the major findings of this study. Also in this section plans are presented for future work.

2 Model equation

Behaviours of many physical systems encountered in models of various mechanisms lead to the telegraph equation. The following one-dimensional telegraph equation, arising in various fields of science, is

$$u_{tt} + \alpha u_t + \beta u = c^2 u_{xx} + h(x,t), \qquad 0 < x < L, \qquad 0 < t \leq T \tag{1}$$

$$u_{tt} + \alpha u_t + \beta u = c^2 u_{xx} + h(x,t), \qquad 0 < x < L, \qquad 0 < t \leq T, \tag{1}$$

with the initial conditions

$$u(x,0) = g_1(x), \qquad 0 < x < L, \tag{2}$$

$$u_t(x,0) = g_2(x), \qquad 0 < x < L, \tag{3}$$

where α and β are known constant coefficients, h,g_1,g_2 are known functions and the unknown function u can be voltage or current through the wire at position x and time t. Equation (1) refers to as second-order telegraph equation with constant coefficients. In equation (1) $\alpha = \frac{G}{C} + \frac{R}{L}$, $\beta = \frac{GR}{CL}$ and $c^2 = \frac{1}{LC}$, where G is conductance of resistor, R is resistance of resistor, L is inductance of coil, and C is capacitance of capacitor.

3 The new iteration method

Let us consider the following general equation [14]:

$$u(x) = N(u(x)) + f(x) \tag{4}$$

where N is an operator and f is a known function $x = (x_1, x_2, ..., x_n)$. It is being looked for a solution u of equation (4) having the series form

$$u(x) = \sum_{i=0}^{\infty} u_i(x). \tag{5}$$

The operator N can be decomposed as:

$$N(\sum_{i=0}^{\infty} u_i) = N(u_0) + \sum_{i=1}^{\infty}\{N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j)\}. \tag{6}$$

Use of equations (5) and (6) makes equation (4)

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty}\{N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j)\}. \tag{7}$$

The recurrence relation is defined as:

$$\begin{cases} u_0 = f \\ u_1 = N(u_0) \\ u_{n+1} = N(u_0 + ... + u_n) - N(u_0 + ... + u_{n-1}), n = 1, 2, ... \end{cases} \tag{8}$$

Thus

$$(u_1 + ... + u_{n+1}) = N(u_0 + ... + u_n), \qquad n = 1, 2, ... \tag{9}$$

and

$$\sum_{i=0}^{\infty} u_i = f + N(\sum_{i=0}^{\infty} u_i). \tag{10}$$

The k-term approximate solution of (4) and (5) is given by $u = u_0 + u_1 + ... + u_{k-1}$.

3.1 Application of the method to the telegraph equation

For solving the telegraph equation by the new iteration method, according to the initial conditions consider operator $L_{tt} = \frac{\partial^2}{\partial t^2}$. The inverse operator of L_{tt} is $L_{tt}^{-1} = \int_0^t \int_0^t (.)dtdt$. Applying the inverse operator to both sides of equation (1) gives the following integral equation:

$$u(x,t) = u(x,0) + \frac{\partial u(x,0)}{\partial t} + \int_0^t \int_0^t h(x,t)dtdt + \int_0^t \int_0^t (c^2 \frac{\partial^2 u}{\partial x^2} - \alpha \frac{\partial u}{\partial t} - \beta u)dtdt. \tag{11}$$

Let $N(u) = \int_0^t \int_0^t (c^2 \frac{\partial^2 u}{\partial x^2} - \alpha \frac{\partial u}{\partial t} - \beta u)dtdt.$

Implementation of the DGJ algorithm gives

$$u_0 = g_1 + g_2 t + \int_0^t \int_0^t h(x,t)dtdt,$$

$$u_1 = N(u_0) = \int_0^t \int_0^t (c^2 \frac{\partial^2 u_0}{\partial x^2} - \alpha \frac{\partial u_0}{\partial t} - \beta u_0)dtdt.$$

$$u_{m+1} = N(u_0 + u_1 + ... + u_m) - N(u_0 + u_1 + ... + u_{m-1}), \qquad m = 1, 2, ...$$

4 Convergence of the Method

The condition for convergence of the series $\sum u_i$ is presented below. For the details the reader is referred to [14].

Theorem 4.1. *If N is $C^{(\infty)}$ in a neighborhood of u_0 and $\|N^{(n)}(u_0)\| \leq L$, for any n and for some real $L > 0$ and $\|u_i\| \leq M < e^{-1}$, $i = 1, 2, ...$ then the series $\sum_{i=1}^{\infty} G_n$ is absolutely convergent and moreover,*

$$\|G_n\| \leq LM^n e^{n-1}(e-1), \qquad n = 1, 2, ...$$

Theorem 4.2. *If N is $C^{(\infty)}$ and $\|N^{(n)}(u_0)\| \leq M < e^{-1}$, $\forall n$, then the series $\sum_{i=1}^{\infty} G_n$ is absolutely convergent.*

5 Illustrative examples

Now, the proposed technique is applied to solve some examples of different forms of the telegraph equation. Note also that less computation is needed in the proposed method comparison to the conventional iterative methods. To verify the efficiency of the present method for the current problem in comparison with the literature in terms of exact solution, absolute and relative errors. Consider the hyperbolic telegraph equation in the form (1) with the initial conditions (2) and (3). All approximate solutions were obtained using MATLAB 7.0. As various problems of science were modelled by the partial differential equations and since therefore the hyperbolic telegraph equation is of high importance, the following examples are selected. Note that the exact solutions have been obtained in the first two examples. And then, for the last two examples, the absolute and relative errors are presented in Tables 1 and 2, respectively, to show that how the series rapidly converge to the exact solution.

Example 1. *[2,11] Consider the telegraph equation (1) with the following initial conditions;*

$$u(x,0) = e^x,$$

$$\frac{\partial u(x,0)}{\partial t} = -e^x,$$

For the example, the equation is solved for $\alpha = \beta = c = 1$ and $h(x,t) = 0$.

Implementation of the DGJ algorithm gives

$$u_0 = e^x(1-t),$$

$$u_1 = \frac{1}{2}e^x t^2,$$

$$u_2 = -\frac{1}{6}e^x t^3,$$

$$u_3 = \frac{1}{24}e^x t^4,$$

$$u_4 = \frac{1}{120}e^x t^5,$$

$$\vdots$$

$$u_n = \frac{1}{(n+1)!}e^x t^{n+1},$$

$$\vdots$$

When $n \to \infty$, the result converges to exact solution as follows:

$$u(x,t) = lim_{n\to\infty}\sum_{k=0}^{n} u_k = e^x lim_{n\to\infty}\sum_{k=0}^{n}\frac{(-t)^k}{k!} = e^{x-1}.$$

Example 2. *[2] Consider the hyperbolic telegraph equation (1) in the interval $0 \le x \le \pi$. The initial conditions are given by*

$$u(x,0) = g_1(x) = \sin(x), \qquad 0 \le x \le \pi$$

$$u_t(x,0) = g_2(x) = -\sin(x), \qquad 0 \le x \le \pi$$

The analytical solution is given in [1] as

$$u(x,t) = \exp(-t)\sin(x).$$

For the present example, the equation is solved for $\alpha = 4$, $\beta = 2$ *and* $c = 1$ *. For this case we have*

$$h(x,t) = (2-\alpha+\beta)\exp(-t)\sin(x).$$

By beginning with $u_0 = \sin(x) - \sin(x)t$, *then in the same manner we will have:*

$$u_1 = \frac{1}{2}\sin(x)(t^2+t^3),$$

$$u_2 = \sin(x)(-\frac{2}{3}t^3 - \frac{5}{8}t^4 - \frac{3}{40}t^5),$$

$$u_3 = \sin(x)(\frac{2}{3}t^4 + \frac{3}{5}t^5 + \frac{9}{80}t^6 + \frac{3}{560}t^7),$$

$$u_4 = \sin(x)(-\frac{8}{15}t^5 - \frac{7}{15}t^6 - \frac{3}{28}t^7 - \frac{39}{4480}t^8 - \frac{1}{4480}t^9),$$

$$\vdots$$

When $n \to \infty$ *, the* n*-terms solution convergences to the analytical solution, namely,*

$$u(x,t) = \sum_{k=0}^{n} u_k = \sin(x)(1 - t + \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{24}t^4 - \frac{1}{120}t^5 + \frac{1}{720}t^6 - ...)$$
$$= \sin(x)\exp(-t).$$

Example 3. *[11] Consider the telegraph equation (1) with the following initial conditions,*

$$u(x,0) = \sin x,$$

$$u_t(x,0) = 0.$$

By using the iteration method (for $\alpha = 2$, $\beta = c = 1$ *six-terms approximation to the solution will be as follows) we have;*

$$u_0 = \sin x,$$

$$u_1 = -t^2 \sin x,$$

$$u_2 = \frac{2}{3}t^3\sin x + \frac{1}{6}t^4 \sin x,$$

$$u_3 = -\frac{1}{3}t^4\sin x - \frac{2}{15}t^5\sin x - \frac{1}{90}t^6\sin x,$$

$$u_4 = \frac{2}{15}t^5\sin x + \frac{1}{15}t^6\sin x + \frac{1}{105}t^7\sin x + \frac{1}{2520}t^8\sin x,$$

$$u_5 = -\frac{2}{45}t^6\sin x - \frac{8}{315}t^7\sin x - \frac{1}{210}t^8\sin x - \frac{1}{2835}t^9\sin x - \frac{1}{113400}t^{10}\sin x,$$

$$u_4 = ...$$

and then

$$u(x,t) = \sin x(1 - t^2 + \frac{2}{3}t^3 - \frac{1}{6}t^4 + \frac{1}{90}t^6 - \frac{1}{63}t^7 - \frac{11}{2520}t^8 - \frac{1}{2835}t^9 - \frac{1}{113400}t^{10} + ...).$$

This result is exactly in agreement with the result of Biazar et al. [11].

Example 4. *[11] Consider the telegraph equation (1) with the following initial conditions;*

$$u(x,0) = e^x,$$

$$\frac{\partial u(x,0)}{\partial t} = x,$$

$$\alpha = 5,\ \beta = 6,\ c = 4.$$

Implementation of the DGJ algorithm leads to

$$u_0 = e^x + xt,$$

$$u_1 = \frac{1}{2}(10e^x - 5x)t^2 - xt^3,$$

$$u_2 = (\frac{25x}{6} - \frac{25e^x}{3})t^3 + (\frac{5x}{2} + \frac{25e^x}{6})t^4 + \frac{3}{10}xt^5,$$

$$u_3 = (\frac{125e^x}{12} - \frac{10x}{3})t^4 + (-\frac{25e^x}{3} - \frac{15x}{4})t^5 + (\frac{25e^x}{18} - \frac{3x}{4})t^6 - \frac{3}{70}xt^7,$$

$$u_4 = \frac{x}{280}t^9 + (\frac{3x}{28} - \frac{125e^x}{504})t^8 + (\frac{15x}{14} - \frac{125e^x}{42})t^7 + (\frac{25x}{6} + \frac{125e^x}{12})t^6 + (\frac{125x}{24} - \frac{125e^x}{12})t^5,$$

$$\vdots$$

$$u = e^x + xt + (-\frac{5x}{2} + 5e^x)t^2 + (\frac{19x}{6} - \frac{25e^x}{3})t^3 + (-\frac{65x}{24} + \frac{175e^x}{12})t^4$$

$$+(\frac{211x}{120} - \frac{75e^x}{4})t^5 + (\frac{41x}{12} + \frac{425e^x}{36})t^6 + (\frac{36x}{35} - \frac{125e^x}{42})t^7 + (\frac{3x}{28} - \frac{125e^x}{504})t^8 + \frac{x}{280}t^9 + \ldots$$

This result is exactly in agreement with the result of Biazar et al. [11].

Example 5. *[2] Similar to previous examples, we consider the hyperbolic telegraph equation (1) with* $\alpha = 1$, $\beta = 1$, $c = 1$ *and* $h(x,t) = x^2 + t - 1$ *in the interval* $0 \le x \le 1$ *. The initial conditions are:*

$$u(x,0) = g_1(x) = x^2, \qquad 0 \le x \le 1$$

$$u_t(x,0) = g_2(x) = 1, \qquad 0 \le x \le 1$$

The exact solution is

$$u(x,t) = x^2 + t.$$

By beginning with

$$u_0 = x^2 + t + \frac{1}{2}x^2t^2 + \frac{1}{6}t^3 - \frac{1}{2}t^2,$$

then in the same manner we will have:

$$u_1 = \frac{1}{2}t^2 + \frac{1}{12}t^4 - \frac{1}{6}x^2t^3 - \frac{1}{2}x^2t^2 - \frac{1}{24}x^2t^4 - \frac{1}{120}t^5,$$

$$u_2 = -\frac{1}{30}t^5 - \frac{1}{240}t^6 - \frac{1}{8}t^4 + \frac{1}{12}x^2t^4 + \frac{1}{60}x^2t^5 - \frac{1}{6}t^3 + \frac{1}{720}x^2t^6 + \frac{1}{5040}t^7 + \frac{1}{6}x^2t^3,$$

$$u_3 = \frac{11}{720}t^6 + \frac{11}{5040}t^7 + \frac{1}{10080}t^8 + \frac{1}{20}t^5 - \frac{1}{40}x^2t^5 - \frac{1}{180}x^2t^6 - \frac{1}{1680}x^2t^7$$

$$-\frac{1}{40320}x^2t^8 - \frac{1}{362800}t^9 + \frac{1}{24}t^4 - \frac{1}{24}x^2t^4,$$

$$u_4 = \frac{1}{368800}x^2t^{10} + \frac{1}{90720}t^9 - \frac{1}{725760}t^{10} - \frac{1}{1344}t^8 - \frac{1}{17280}t^9 - \frac{1}{120}t^5 - \frac{1}{80}t^6 - \frac{23}{5040}t^7$$

$$+\frac{1}{120}x^2t^5 + \frac{1}{180}x^2t^6 + \frac{1}{720}x^2t^7 + \frac{1}{5760}x^2t^8 + \frac{1}{39916800}t^{11},$$

$$\vdots$$

The absolute and relative errors are obtained in Table 1 for $t = 0.5, 1, 5$ and 10 using various number of terms in order to show how the series rapidly converge to the exact solution.

Example 6. *[2] In this example, the hyperbolic telegraph equation (1) is considered with $\alpha = 1$, $\beta = 1$ and $c = 1$ in the interval $0 \le x \le 1$. The initial conditions are:*

$$u(x,0) = g_1(x) = 0, \qquad 0 \le x \le 1,$$

$$u_t(x,0) = g_2(x) = 0, \qquad 0 \le x \le 1$$

and the exact solution [5] is

$$u(x,t) = (x - x^2)t^2 exp(-t).$$

The right hand side function is

$$h(x,t) = (2 - 2t + t^2)(x - x^2)exp(-t) + 2t^2 exp(-t).$$

By starting with

$$u_0 = (x - x^2)(4e^{-t} + t^2e^{-t} + 2te^{-t}) + 2t^2e^{-t} + 8te^{-t} + 12e^{-t} + 2xt - 2x^2t$$
$$+4t - 4x + 4x^2 - 12,$$

then in the same manner we will have

$$u_1 = -44e^{-x} - 2t^2e^{-x} - 16te^{-x} - \tfrac{4}{3}t^3 + 8t^2 + (-x + x^2)(-8e^{-x} - t^2e^{-x}$$
$$-4e^{-x}t) + xt^2 + (-x + x^2)(14e^{-x} + t^2e^{-x} + 6te^{-x}) - \tfrac{1}{3}xt^3 + \tfrac{1}{3}x^2t^3$$
$$-28t - 4xt + 4x^2t - 6x^2 + 6x + 44,$$

$$u_2 = -44 - 2x + 36t + (2x - 2x^2)(-22e^{-x} - t^2e^{-x} - 8te^{-x}) + 2x^2 + (x - x^2)(14e^{-x}$$
$$+t^2e^{-x} + 6te^{-x}) - xt^2 + x^2t^2 + 2xt - 2x^2t + \tfrac{1}{10}t^5 + 44e^{-x} + \tfrac{1}{3}xt^3 - \tfrac{1}{3}x^2t^3$$
$$-\tfrac{1}{2}t^4 + \tfrac{10}{3}t^3 + 8te^{-x} - \tfrac{1}{60}x^2t^5 + (x - x^2)(32e^{-x} + t^2e^{-x} + 10te^{-x}) + \tfrac{1}{60}xt^5 - 14t^2,$$

$$u_3 = 12 - 12t + (-2x + 2x^2)(32e^{-t} + t^2e^{-t} + 10te^{-t}) + (-x + x^2)(-22te^{-t}$$
$$-t^2e^{-t} - 8te^{-t}) - \tfrac{1}{10}t^5 - 12e^{-t} - \tfrac{1}{315}t^7 + (-x + x^2)(-44e^{-t} - t^2e^{-t} - 12te^{-t})$$
$$-\tfrac{1}{360}xt^6 + \tfrac{1}{360}x^2t^6 + \tfrac{1}{2}t^4 - 2t^3 + \tfrac{1}{60}x^2t^5 - \tfrac{1}{60}xt^5 + 6t^2$$
$$+(-2x + 2x^2)(-44e^{-t} - t^2e^{-t} - 12te^{-t}) + (-x + x^2)(58e^{-t} + t^2e^{-t} + 14te^{-t})$$
$$-\tfrac{1}{2520}xt^7 + \tfrac{1}{2520}x^2t^7 + (-x + x^2)(32e^{-t} + t^2e^{-t} + 10te^{-t}),$$

$$u_4 = (-2x + 2x^2)(32e^{-t} + t^2e^{-t} + 10te^{-t}) + (x - x^2)(92te^{-t} + t^2e^{-t} + 18te^{-t})$$
$$+(4x - x^2)(-74e^{-t} - t^2e^{-t} - 16te^{-t}) + (2x - 2x^2)(58e^{-t} + t^2e^{-t} + 14te^{-t})$$
$$+(4x - x^2)(-44e^{-t} - t^2e^{-t} - 12te^{-t}) + (4x - 4x^2)(58e^{-t} + t^2e^{-t} + 14te^{-t})$$
$$-\tfrac{1}{1260}x^2t^7 + \tfrac{1}{1260}xt^7 - (x - x^2)(32e^{-t} + t^2e^{-t} + 10te^{-t}) + \tfrac{1}{315}t^7 - \tfrac{1}{181440}x^2t^9$$
$$+\tfrac{1}{181440}xt^9 + \tfrac{1}{2016}t^8 - \tfrac{1}{10080}x^2t^8 + \tfrac{1}{360}xt^6 + \tfrac{1}{10080}xt^8 - \tfrac{1}{360}x^2t^6 + \tfrac{1}{18144}t^9,$$

$\vdots$

To show how the series rapidly converges to the exact solution, the absolute and relative errors are presented in Table 2 for $t = 0.5, 1, 3$ *and* 5 *using various number of terms.*

6 Conclusions

In this article, implementation of the DGJ algorithm has been explored to obtain analytical approximate solution of the hyperbolic telegraph equation. The method successfully worked to give very reliable results to these processes in the form of analytical approximation converging very rapidly. The results show that the method is seen to be a very reliable alternative and intuitively believed to be a powerful mathematical tool for finding analytical solutions of more intricate linear/nonlinear equations. The method does not need small parameter or linearization as opposed to numerical methods. To illustrate the method and its ability six examples have been presented. The method proves to be applicable and elegant for computer package programs.

Table 2: Convergence rate and accuracy of the results depending on the number of terms used in example 5

Number of u-terms	t	x	u_{exact}	u_{DGJ}	Absolute errors	Relative errors
		0.1	0.01364693984353	0.01364693984421	6.71E-13	4.92E-11
14	0.5	0.5	0.03790816623204	0.03790816621490	1.71E-11	4.52E-10
		0.9	0.01364693984353	0.01364693984421	6.75E-13	4.95E-11
		0.1	0.04032752537797	0.04032752616638	7.88E-10	1.96E-08
21	3.0	0.5	0.11202090382769	0.11202090427697	4.49E-10	4.01E-09
		0.9	0.04032752537797	0.04032752652741	1.15E-09	2.85E-08
		0.1	0.01516038074794	0.01516032372345	5.70E-08	3.76E-06
26	5.0	0.5	0.04211216874428	0.04211210867390	6.01E-08	1.43E-06
		0.9	0.01516038074794	0.01516031844514	6.23E-08	4.11E-06

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Table 1: Convergence rate and accuracy of the results depending on the number of terms used in example 6

Number of u-terms	t	x	u_{exact}	u_{DGJ}	Absolute errors	Relative errors
		0.1	0.510000000	0.510000000	0.00E+00	0.00E+00
	0.5	0.5	0.750000000	0.750000000	0.00E+00	0.00E+00
		0.9	1.310000000	1.310000000	0.00E+00	0.00E+00
		0.1	1.010000000	1.010000000	3.98E-12	3.94E-12
14	1	0.5	1.250000000	1.250000000	3.58E-12	2.87E-12
		0.9	1.810000000	1.810000000	2.66E-12	1.47E-12
		0.1	5.010000000	9.007892343	4.00E+00	7.98E-01
	0.5	0.5	5.250000000	9.094594549	3.84E+00	7.32E-01
		0.9	5.810000000	9.296899697	3.49E+00	6.00E-01
		0.1	0.510000000	0.510000000	0.00E+00	0.00E+00
	0.5	0.5	0.750000000	0.750000000	0.00E+00	0.00E+00
		0.9	1.310000000	1.310000000	0.00E+00	0.00E+00
		0.1	1.010000000	1.010000000	0.00E+00	0.00E+00
	1	0.5	1.250000000	1.250000000	0.00E+00	0.00E+00
		0.9	1.810000000	1.810000000	0.00E+00	0.00E+00
38						
		0.1	5.010000000	5.010000000	0.00E+00	0.00E+00
	5	0.5	5.250000000	5.250000000	0.00E+00	0.00E+00
		0.9	5.810000000	5.810000000	0.00E+00	0.00E+00
		0.1	10.010000000	10.011514344	1.51E-03	1.51E-04
	10	0.5	10.250000000	10.251488228	1.49E-03	1.45E-04
		0.9	10.810000000	10.811427290	1.43E-03	1.32E-04
		0.1	0.510000000	0.510000000	0.00E+00	0.00E+00
	0.5	0.5	0.750000000	0.750000000	0.00E+00	0.00E+00
		0.9	1.310000000	1.310000000	0.00E+00	0.00E+00
		0.1	1.010000000	1.010000000	0.00E+00	0.00E+00
	1	0.5	1.250000000	1.250000000	0.00E+00	0.00E+00
		0.9	1.810000000	1.810000000	0.00E+00	0.00E+00
57						
		0.1	5.010000000	5.010000000	0.00E+00	0.00E+00
	5	0.5	5.250000000	5.250000000	0.00E+00	0.00E+00
		0.9	5.810000000	5.810000000	0.00E+00	0.00E+00
		0.1	10.010000000	10.010000000	0.00E+00	0.00E+00
	10	0.5	10.250000000	10.250000000	0.00E+00	0.00E+00
		0.9	10.810000000	10.810000000	0.00E+00	0.00E+00

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