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Natural Convection Along A Vertical Wavy Surface in A Porous Medium with Variable Properties and Cross Diffusion Effects

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Abstract: In this paper, we study the effect of variable viscosity and variable thermal conductivity on free convective heat and mass transfer along a vertical wavy surface embedded in a Darcy porous medium in the presence of cross diffusion effects. The wavy surface of the vertical plate is transformed into plane geometry case by using a suitable transformation and then solved numerically by employing the Runge-Kutta fourth order method with shooting technique. Numerical results for dimensionless flow velocity, temperature and concentration distribution as well as Nusselt number and Sherwood numbers are presented graphically for various values of Soret and Dufour parameters, variable viscosity, variable thermal conductivity parameters and amplitude of the wavy surface.

Keywords: Wavy Surface; Heat and Mass Transfer; Darcy porous medium; Soret and Dufour effects; Variable Properties

1 Introduction

Natural convection heat and mass transfer from irregular surface has great importance in engineering applications. In many practical situations surfaces are roughened in order to enhance the rate of heat transfer. The presence of roughened surface alters not only the flow field but also alters the heat and mass transfer characteristics. For instance, grain storage containers where walls are buckled and in cavity wall insulating systems. Natural convection from wavy surfaces are used for transferring heat in several heat transfer devices, such as flat plate condensers in refrigerators, flat plate solar collectors, cooling of electrical and nuclear components. Extensive studies of natural convection heat and mass transfer of a vertical wavy surface under boundary layer approximation have been undertaken by several authors. Cheng [1] analyzed double diffusive natural convection along an inclined wavy surface in a porous medium. Mahdy and Ahmed [2] have analyzed laminar free convection over a vertical wavy surface embedded in a porous medium saturated with a Nanofluid. Aziz et. al [3] studied the effect of local thermal non-equilibrium on unsteady heat transfer by natural convection of a nanofluid over a vertical wavy surface.

Most of the previous studies are based on the constant physical properties of the fluid. However it is well known that the fluid property may change with temperature. For many liquids, among them water, petroleum oils, glycerin, silicon fluids, molten salts and some glycols, the viscosity vary with temperature. Therefore, to accurately predict the flow heat and mass transfer rates, it is necessary to take into account the temperature-dependent viscosity and temperature dependent thermal conductivity of the fluid. Variable viscosity and thermal conductivity takes place in many engineering applications such as Heat transfer in furnaces, boilers, porous burners, volumetric solar receivers, fibrous and foam insulations, drawing of plastic films, study of spilling pollutant crude oil over the surface of seawater, in the process of hot cooling, wire drawing, paper production, glass fiber production and gluing of labels on hot bodies. Pal and Mondal [4] used variable viscosity on MHD non-Darcy mixed convective heat transfer over a stretching sheet embedded in a porous medium with non-uniform heat source/sink. Hassanien and Rashed [5] have analyzed non-Darcy free convection flow over a horizontal cylinder in a saturated porous medium with variable viscosity, thermal conductivity and mass diffusivity. Hamad and Uddin et.al [6] analyzed Radiation effects on heat and mass transfer in MHD stagnation point flow over a permeable flat plate with

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thermal convective surface boundary condition, temperature dependent viscosity and thermal conductivity. Vajravelu and Prasad et. al. [7] studied unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable properties.

In most of the studies related to heat and mass transfer process, Soret effect (mass fluxes created by temperature gradients)and Dufour effects (energy flux caused by a concentration gradient) are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fouriers and Ficks laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight and of medium molecular weight. Lakshminarayana and Sibanda [8] have investigated Soret and Dufour effects on free convection along a vertical wavy surface in a fluid saturated Darcy porous medium. Cheng [9] studied Soret and Dufour effects on free convection boundary layers over an inclined wavy surface in a porous medium. Rathish Kumar and Krishna Murthy [10] investigated double diffusive free convection induced by vertical wavy surface in a doubly stratified Darcy porous medium under the influence of Soret and Dufour effect. These effects may become significant in different areas such as petrology or Geo-sciences and hydrology.

The aim of the present paper is to investigate the effect of variable viscosity and variable thermal conductivity along with Soret and Dufour effects on natural convective heat and mass transfer over a vertical wavy surface embedded in a fluid saturated porous medium. The governing equations are converted into a set of coupled differential equations by using the boundary layer transformation [11], and the obtained equations are solved by employing Runge-Kutta fourth order method with shooting technique. The effects of variable viscosity, variable thermal conductivity, Soret parameter, Dufour parameter and dimensionless amplitude on heat and mass transfer characteristics at a vertical wavy surface in a porous medium are examined and results are presented graphically. The results obtained are compared with previously published work.

2 Formulation of the problem

Consider the boundary layer flow near the wavy surface in a fluid saturated porous medium as shown in fig-1. The wavy surface is described by

$$y^* = \sigma^*(x^*) = a^* sin\left(\frac{\pi x^*}{l}\right)$$

where a^* is the amplitude of the wavy surface, and l is the characteristic length of the wavy surface. The wavy surface is held at constant temperature T_w and constant concentration C_w which are higher than the porous medium temperature T_∞ and concentration C_∞ sufficiently far from the wavy surface. The governing equations for this problem under the Boussinesq approximations are given by (See ([15]))

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial}{\partial y^*}\left(\frac{\mu}{K}u^*\right) = \frac{\partial}{\partial x^*}\left(\frac{\mu}{K}v^*\right) + \rho g\beta_t\frac{\partial T}{\partial y^*} + \rho g\beta_c\frac{\partial C}{\partial y^*} \tag{2}$$

$$u^*\frac{\partial T}{\partial x^*} + v^*\frac{\partial T}{\partial y^*} = \frac{\partial}{\partial x^*}\left(\alpha\frac{\partial T}{\partial x^*}\right) + \frac{\partial}{\partial y^*}\left(\alpha\frac{\partial T}{\partial y^*}\right) + \frac{Dk_T}{c_s c_p}\left(\frac{\partial^2 C}{\partial (x^*)^2} + \frac{\partial^2 C}{\partial (y^*)^2}\right) \tag{3}$$

$$u^*\frac{\partial C}{\partial x^*} + v^*\frac{\partial C}{\partial y^*} = D\left(\frac{\partial^2 C}{\partial (x^*)^2} + \frac{\partial^2 C}{\partial (x^*)^2}\right) + \frac{Dk_T}{T_m}\left(\frac{\partial^2 T}{\partial (x^*)^2} + \frac{\partial^2 T}{\partial (y^*)^2}\right) \tag{4}$$

where u^* and v^* are the volume averaged velocity components in x^* and y^* directions, respectively. μ is the dynamic coefficient of viscosity of the fluid, ρ is the density, K is the permeability of the porous medium, β_t is the coefficients of thermal expansion, β_c is the coefficient of concentration expansion, α is the dimensional thermal conductivity, k_T is the thermal diffusion ratio, D is the mass diffusivity of the saturated porous medium, c_s is the concentration susceptibility, c_p is the specific heat at constant pressure, T_m is the mean fluid temperature and g is the gravitational acceleration. The corresponding boundary conditions are

$$\left.\begin{array}{l} u^* = 0, \quad v^* = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y^* = \sigma^*(x^*) = a^* sin\left(\frac{\pi x^*}{l}\right) \\ u^* \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y^* \to \infty \end{array}\right\} \tag{5}$$

The fluid properties are assumed to be isotropic and constant, except for the fluid viscosity μ and fluid thermal conductivity α. The fluid viscosity μ which is assumed to vary as be an inverse linear function of the temperature T ([12]) as

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left(1 + \delta(T - T_\infty)\right) \quad or \quad \frac{1}{\mu} = b(T - T_r) \tag{6}$$

where $b = \frac{\delta}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\delta}$. Both b and T_r are constants and their values depend on the reference state and the thermal property of the fluid i.e. δ. In general $b > 0$ for liquids and $b < 0$ for gases. The variable viscosity parameter θ_r, which is defined by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\delta(T_w - T_\infty)}$$

is constant. It is an important to note that for $\delta \to 0$ (i.e. $\mu = \mu_\infty$=constant) then $\theta_r \to \infty$. It is also mentioning here that θ_r is positive for gases and negative for liquids. T_∞ is the free stream temperature. Also, we assume that the fluid thermal conductivity α is assumed to vary as a linear function of temperature in the form ([13]):

$$\alpha = \alpha_o\left(1 + E(T - T_\infty)\right)$$

where α_o is the thermal diffusivity at the wavy surface temperature T_w and E is a constant depending on the nature of the fluid. It is worth mentioning here that $E < 0$ for fluids such as lubrication oils, while $E > 0$ for fluids such as air, water. This can be written in the non-dimensional form ([14]) as

$$\alpha = \alpha_o(1 + \beta\theta) \tag{7}$$

where $\beta = E(T_w - T_\infty)$ is the thermal conductivity parameter and T_w is the wavy surface temperature. The variation of β can be taken in the range as $-0.1 \le \beta \le 0$ for lubrication oils, $0 \le \beta \le 0.12$ for water and $0 \le \beta \le 6$ for air. Here we introduce the stream function ψ^* such that

$$u^* = \frac{\partial\psi^*}{\partial y^*}, \quad v^* = -\frac{\partial\psi^*}{\partial x^*}$$

Introducing the following non-dimensional variables

$$\left.\begin{array}{l} (x, y, a, \sigma) = (x^*, y^*, a^*, \sigma^*)/l, \quad \overline{\psi} = \psi^*/\alpha_o \\ \theta = \dfrac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \dfrac{C - C_\infty}{C_w - C_\infty}, \end{array}\right\} \tag{8}$$

into Eqns. (2)-(4), we get

$$\frac{1}{\theta - \theta_r}\left(\frac{\partial\theta}{\partial y}\frac{\partial\overline{\psi}}{\partial y} + \frac{\partial\theta}{\partial x}\frac{\partial\overline{\psi}}{\partial x}\right) + \frac{\partial^2\overline{\psi}}{\partial y^2} + \frac{\partial^2\overline{\psi}}{\partial x} = Ra\left(1 - \frac{\theta}{\theta_r}\right)\left(\frac{\partial\theta}{\partial y} + N\frac{\partial\phi}{\partial y}\right) \tag{9}$$

$$\frac{\partial\overline{\psi}}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\overline{\psi}}{\partial x}\frac{\partial\theta}{\partial y} = \beta\left(\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right) + (1 + \beta\theta)\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + Du\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) \tag{10}$$

$$\frac{\partial\overline{\psi}}{\partial y}\frac{\partial\phi}{\partial x} - \frac{\partial\overline{\psi}}{\partial x}\frac{\partial\phi}{\partial y} = \frac{1}{Le}\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + Sr\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) \tag{11}$$

where $Ra = \frac{g\beta_t K(T_w - T_\infty)l}{\alpha_o\nu}$ is the Darcy-Rayleigh number, $\nu = \frac{\mu_\infty}{\rho}$ is the kinematic viscosity of the fluid, $N = \frac{\beta_c(C_w - C_\infty)}{\beta_t(T_w - T_\infty)}$ is the buoyancy ratio, $Le = \frac{\alpha_o}{D}$ is the Lewis number, $Du = \frac{Dk_T\triangle C}{\alpha_0 c_s c_p\triangle T}$ is Dufour parameter and $Sr = \frac{Dk_T\triangle T}{\alpha_0 T_m\triangle C}$ is the Soret parameter. The corresponding boundary conditions are given by

$$\left.\begin{array}{llll} \overline{\psi} = 0, \quad \theta = 1, \quad \phi = 1 & \text{on} & y = asin(x), \\ \overline{\psi}_y \to 0, \quad \theta \to 0, \quad \phi \to 0 & \text{as} & y \to \infty. \end{array}\right\} \tag{12}$$

We transfer the effect of the wavy surface from the boundary conditions into the governing equations by the following group of coordinate transformation given by

$$x = \xi, \quad y = \xi^{1/2}Ra^{-1/2}\eta + asin(x), \quad \overline{\psi} = Ra^{1/2}\psi. \tag{13}$$

In order to obtain the following boundary layer equations we substitute Eq. (13) into Eqns.(9)-(11) and letting $Ra \to \infty$ (i.e., boundary layer approximation).

$$\frac{1}{\theta-\theta_r}\left(1+a^2cos^2\xi\right)\frac{\partial\theta}{\partial\eta}\frac{\partial\psi}{\partial\eta}+\left(1+a^2cos^2\xi\right)\frac{\partial^2\psi}{\partial\eta^2}=\xi^{1/2}\left(1-\frac{\theta}{\theta_r}\right)\left(\frac{\partial\theta}{\partial\eta}+N\frac{\partial\psi}{\partial\eta}\right) \tag{14}$$

$$\xi^{1/2}\left(\frac{\partial\psi}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial\psi}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right)=\beta\left(1+a^2cos^2\xi\right)\left(\frac{\partial\theta}{\partial\eta}\right)^2+(1+\beta\theta)\left(1+a^2cos^2\xi\right)\frac{\partial^2\theta}{\partial\eta^2}+Du\left(1+a^2cos^2\xi\right)\frac{\partial^2\phi}{\partial\eta^2} \tag{15}$$

$$\xi^{1/2}\left(\frac{\partial\psi}{\partial\eta}\frac{\partial\phi}{\partial\xi}-\frac{\partial\psi}{\partial\xi}\frac{\partial\phi}{\partial\eta}\right)=\frac{1}{Le}\left(1+a^2cos^2\xi\right)\frac{\partial^2\phi}{\partial\eta^2}+Sr\left(1+a^2cos^2\xi\right)\frac{\partial^2\theta}{\partial\eta^2} \tag{16}$$

For the system of ordinary differential equations, we introduce the following similarity transformations into (14)-(16)

$$\widehat{\eta}=\frac{\eta}{1+a^2cos^2\xi},\quad \psi=\xi^{1/2}f(\widehat{\eta}),\quad \theta=\theta(\widehat{\eta})\quad\text{and}\quad \phi=\phi(\widehat{\eta}) \tag{17}$$

we obtain

$$f''+\frac{1}{\theta-\theta_r}\theta'f'=\left(1-\frac{\theta}{\theta_r}\right)(\theta'+N\phi') \tag{18}$$

$$\beta(\theta')^2+(1+\beta\theta)\theta''+\frac{1}{2}f\theta'+Du\phi''=0 \tag{19}$$

$$\frac{1}{Le}\phi''+\frac{1}{2}f\phi'+Sr\theta''=0 \tag{20}$$

subject to the boundary conditions

$$\left.\begin{array}{llll} f=0, & \theta=1, & \phi=1 & \text{at}\quad \widehat{\eta}=0,\\ f'\to 0, & \theta\to 0, & \phi\to 0 & \text{as}\quad \widehat{\eta}\to\infty \end{array}\right\} \tag{21}$$

where prime denotes differentiation with respect to $\widehat{\eta}$. When $\theta_r \to \infty$ and $\beta = 0$ the problem is equivalent to Lakshminaraya et.al. [8].

The rate of heat transfer (local Nusselt number) and rate of mass transfers (local Sherwood number) are defined in terms of Ra_x and the wave amplitude function a as

$$Nu_x=\frac{-\theta'(0)Ra_x^{1/2}}{(1+a^2cos^2(x))^{1/2}}\quad Sh_x=\frac{-\phi'(0)Ra_x^{1/2}}{(1+a^2cos^2(x))^{1/2}} \tag{22}$$

3 Results and Discussion

The a set of nonlinear non-homogeneous differential equation (18)-(20) with corresponding boundary conditions (21) are solved numerically using a shooting technique along with fourth order Runge-Kutta integration.

In order to assess the accuracy of the present numerical method, we compared our results with those of Cheng [15] in the absence of fluid viscosity parameter, thermal conductivity parameter, Soret and Dufour parameter. The comparison in the above case is found to be in good agreement, as shown in Table. (1)

The effect of various parameters on velocity, temperature and concentration fields have been presented in Figs.(2)-(23) and analyzed. The effect of variable viscosity θ_r on the velocity, temperature and concentration profiles with respect to η is shown in Figs.(2)-(4). With increasing variable viscosity parameter (θ_r), the fluid boundary layer, thermal and solutal boundary layer thickness gradually reduced, which in turns causes to decrease velocity, temperature and concentration profiles. This can be explained physically as the parameter (θ_r) increases, the fluid viscosity increases resulting the depreciation in the boundary layer thickness.

The effect of thermal conductivity parameter β on the velocity, temperature and concentration profiles with respect to η are given in Figs.(5)-(7). Fig.(5) shows that increasing thermal conductivity parameter retards the flow considerably and hence it reduces the velocity boundary layer. From Fig.(6), it is clear that increase the values of β tends to increase the temperature profile due to increase in thermal boundary layer thickness. Fig.(7) reveals that with increasing β, concentration profile is found to decrease, that is β causes to reduce solutal boundary layer thickness

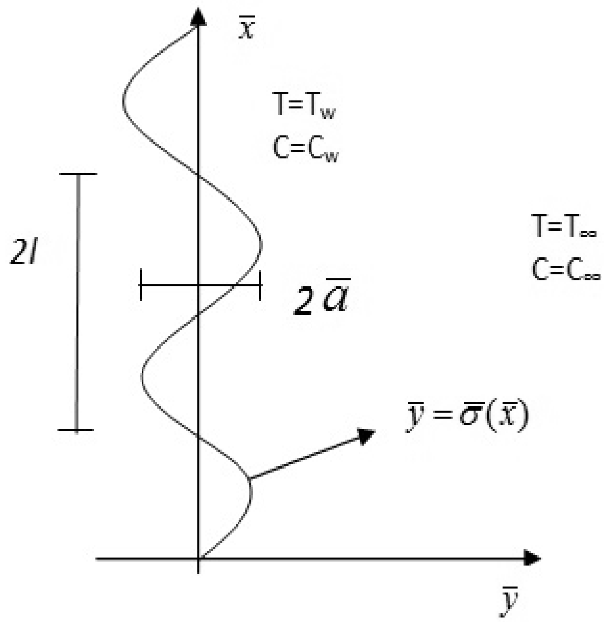

Figure 1: Physical model and coordinates

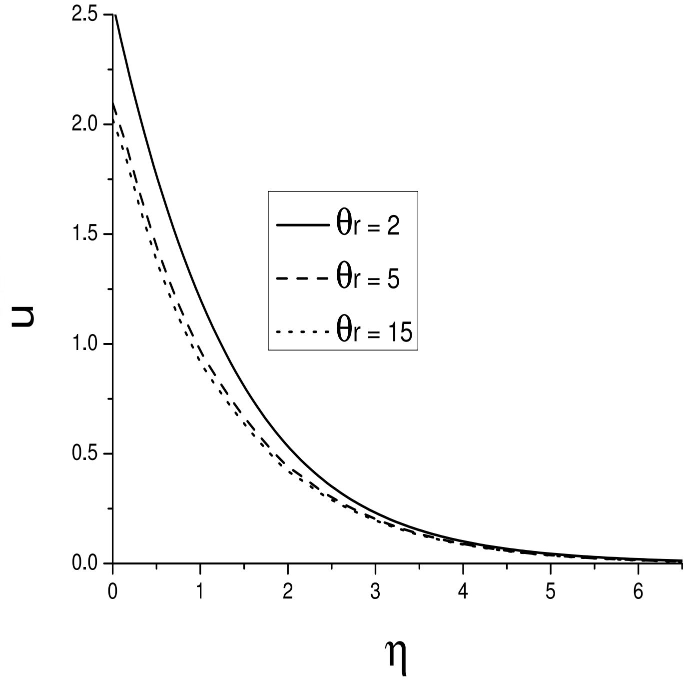

Figure 2: velocity profile for different values of θ_r for $\beta = 0.5, N = 1, Le = 1, Sr = 1$ and $Du = 0.1$

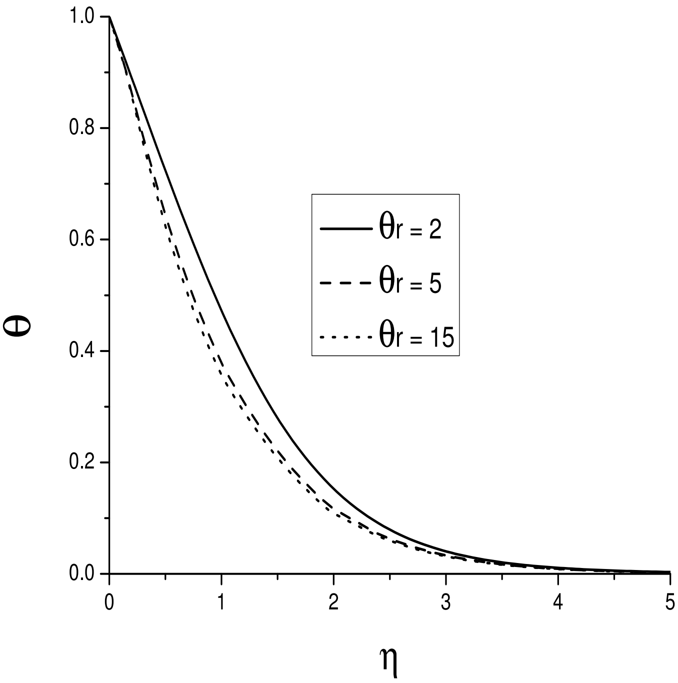

Figure 3: Temperature profile for different values of θ_r for $\beta = 0.5, N = 1, Le = 1, Sr = 1$ and $Du = 0.1$

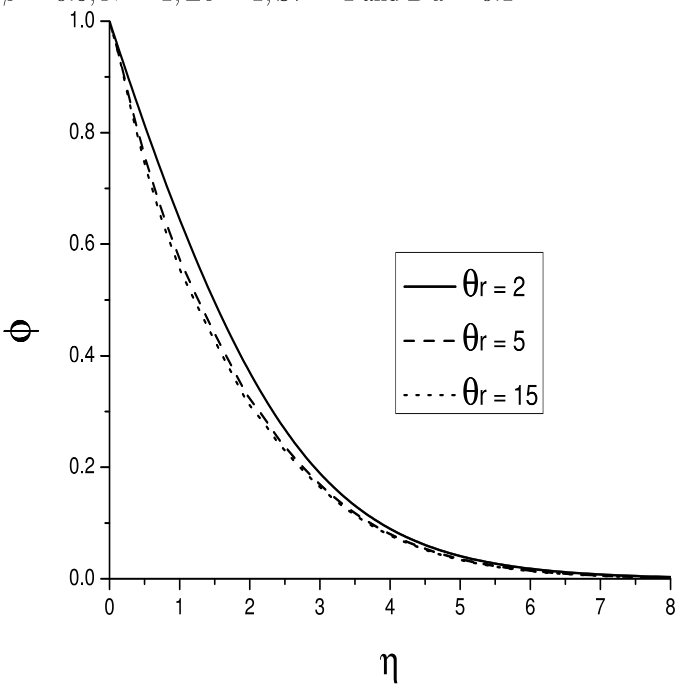

Figure 4: Concentration profile for different values of θ_r for $\beta = 0.5, N = 1, Le = 1, Sr = 1$ and $Du = 0.1$

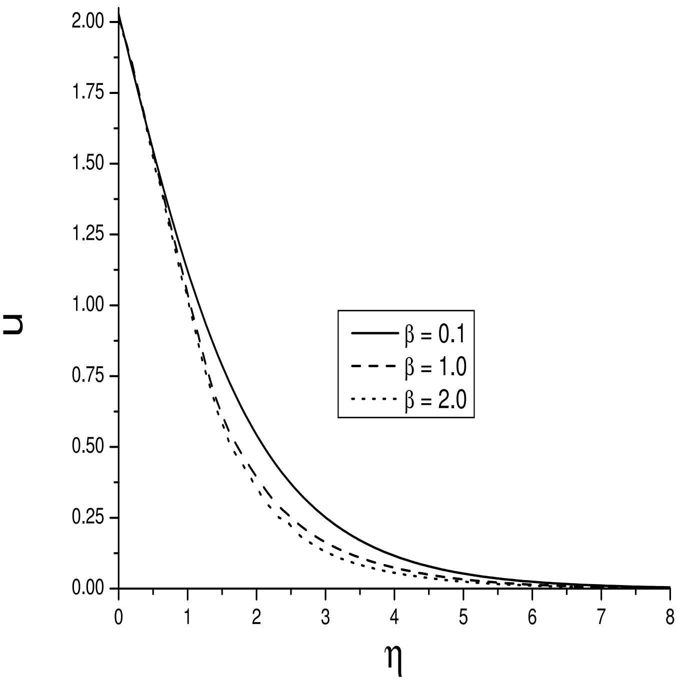

Figure 5: Velocity profile for different values of β for $\theta_r = 15, N = 1, Le = 1, Sr = 1$ and $Du = 0.1$

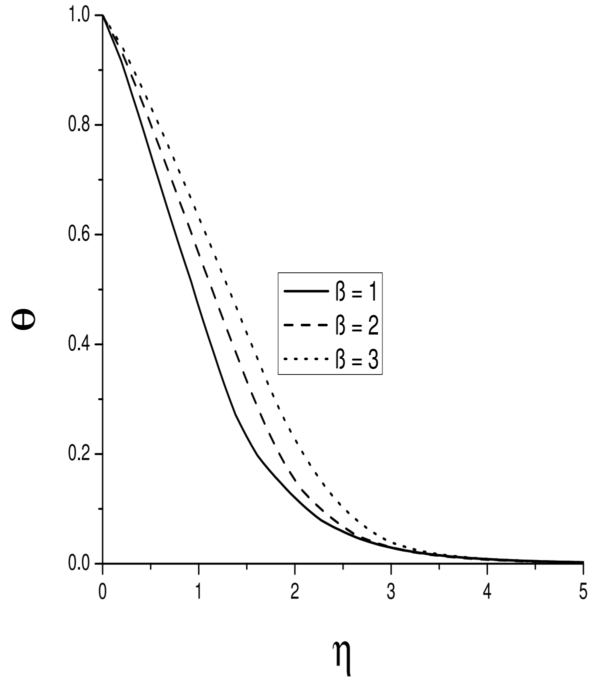

Figure 6: Temperature profile for different values of β for $\theta_r = 15, N = 1, Le = 1, Sr = 1$ and $Du = 0.1$

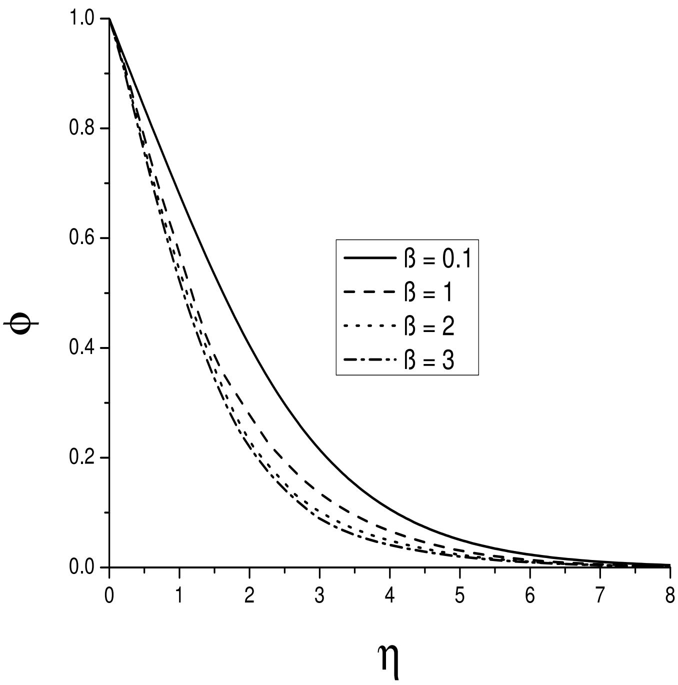

Figure 7: Concentration profile for different values of β for $\theta_r = 15, N = 1, Le = 1, Sr = 1$ and $Du = 0.1$

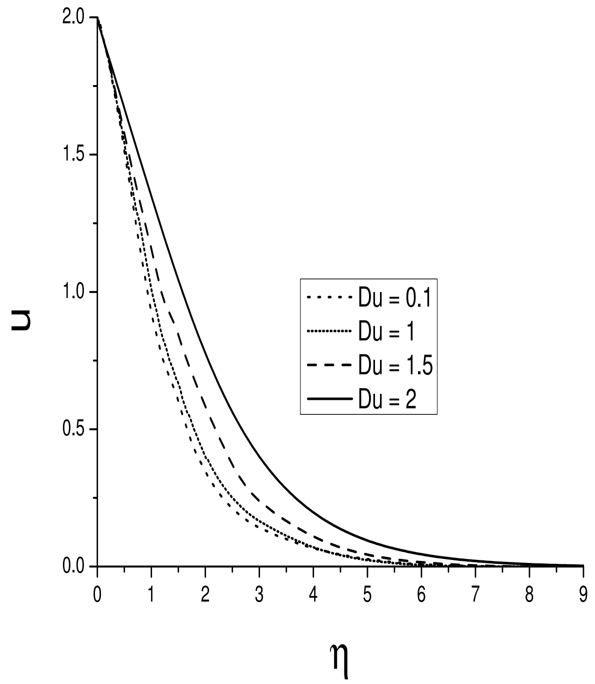

Figure 8: Velocity profile for different values of Du for $\theta_r = 15, N = 1, Le = 1, Sr = 1$ and $\beta = 0.5$

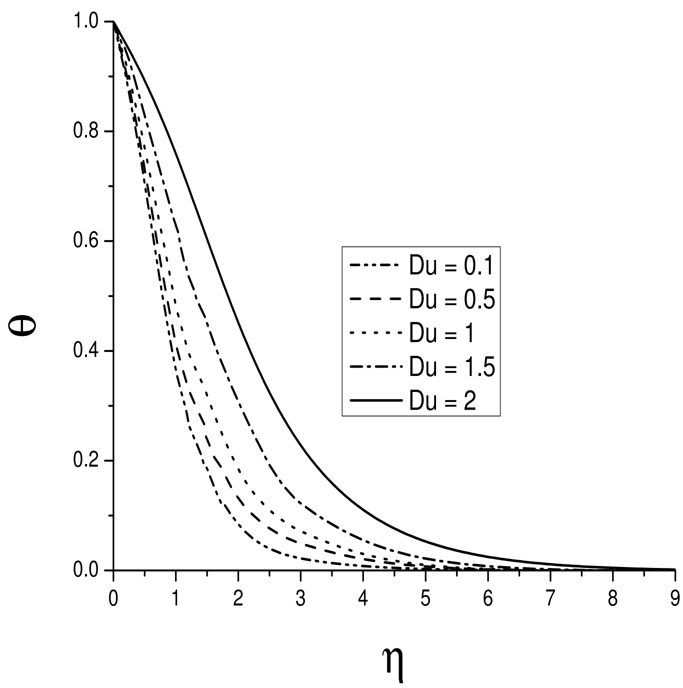

Figure 9: Temperature profile for different values of Du for $\theta_r = 15, N = 1, Le = 1, Sr = 1$ and $\beta = 0.5$

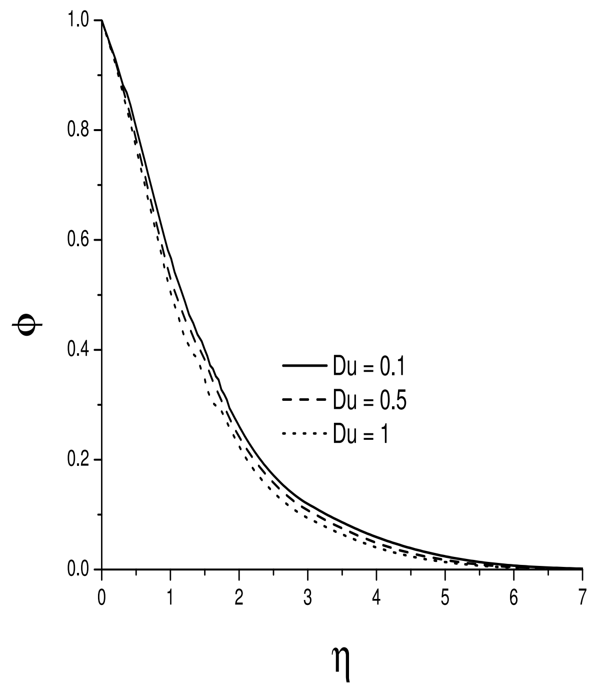

Figure 10: Concentration profile for different values of Du for $\theta_r = 15, N = 1, Le = 1, Sr = 1$ and $\beta = 0.5$

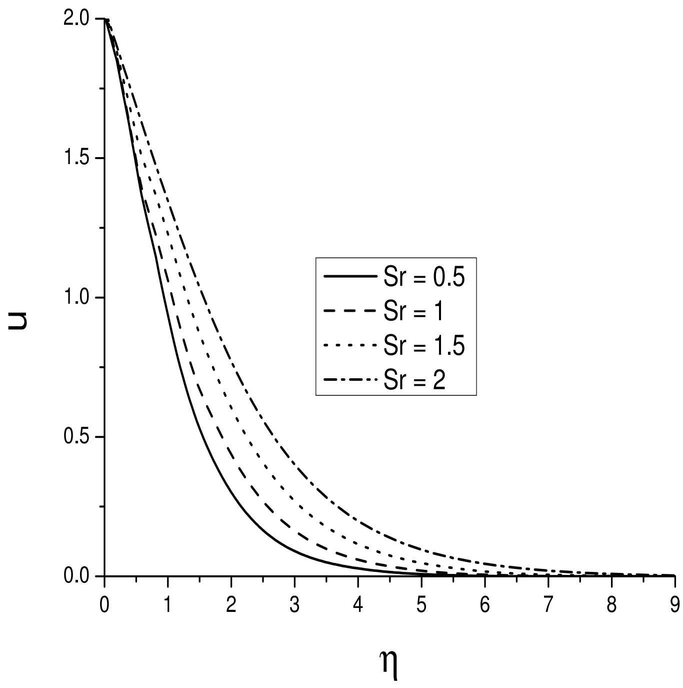

Figure 11: Velocity profile for different values of Sr for $\theta_r = 15, N = 1, Le = 1, Du = 0.1$ and $\beta = 0.5$

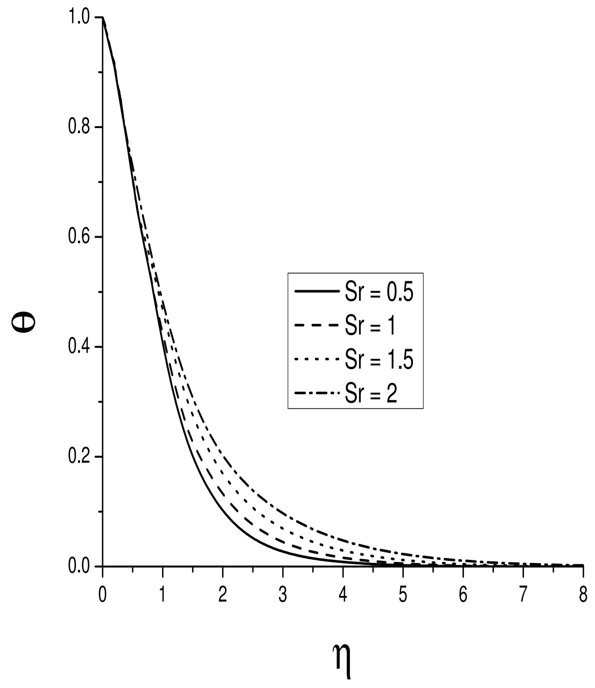

Figure 12: Temperature profile for different values of Sr for $\theta_r = 15, N = 1, Le = 1, Du = 0.1$ and $\beta = 0.5$

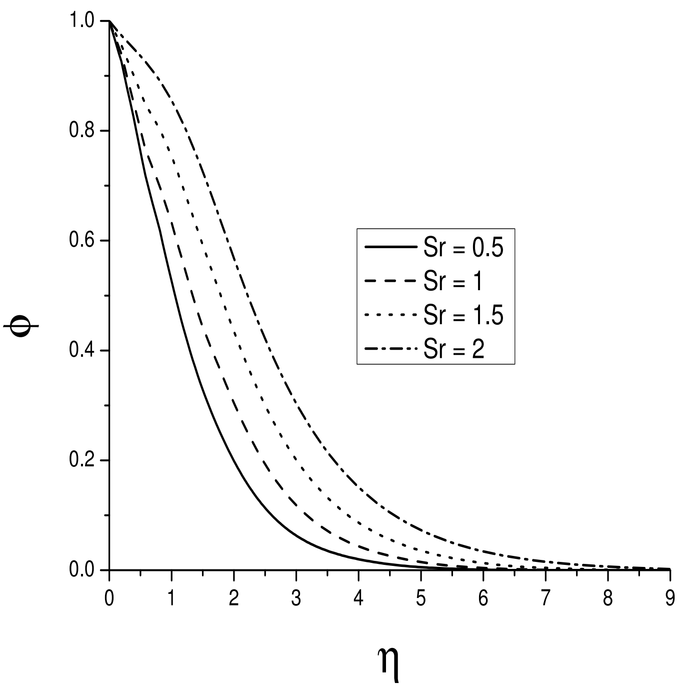

Figure 13: Concentration profile for different values of Sr for $\theta_r = 15, N = 1, Le = 1, Du = 0.1$ and $\beta = 0.5$

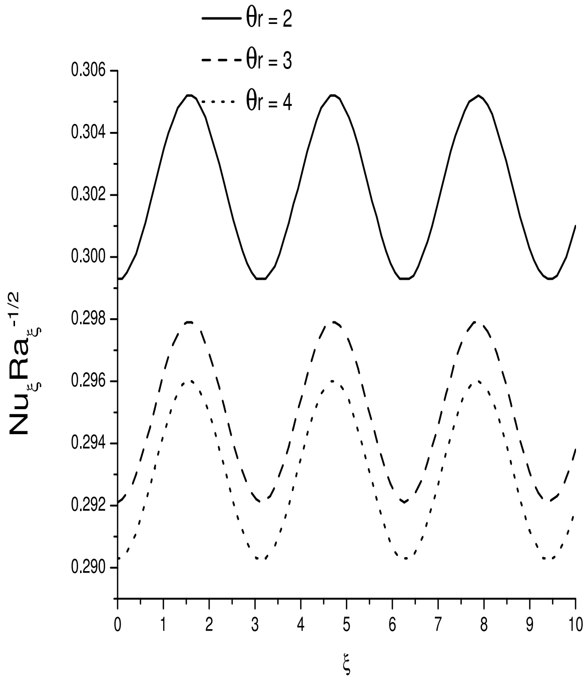

Figure 14: Local Nusselt number verses ξ for various values of θ_r for $Sr = 1, N = 1, Le = 1, Du = 0.1, a = 0.5$ and $\beta = 0.5$

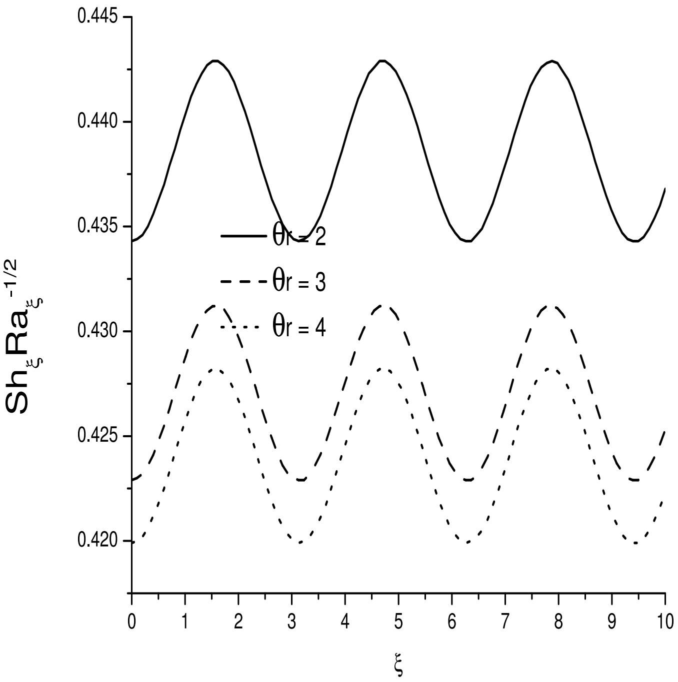

Figure 15: Local Sherwood number verses ξ for various values of θ_r for $Sr = 1, N = 1, Le = 1, Du = 0.1, a = 0.5$ and $\beta = 0.5$

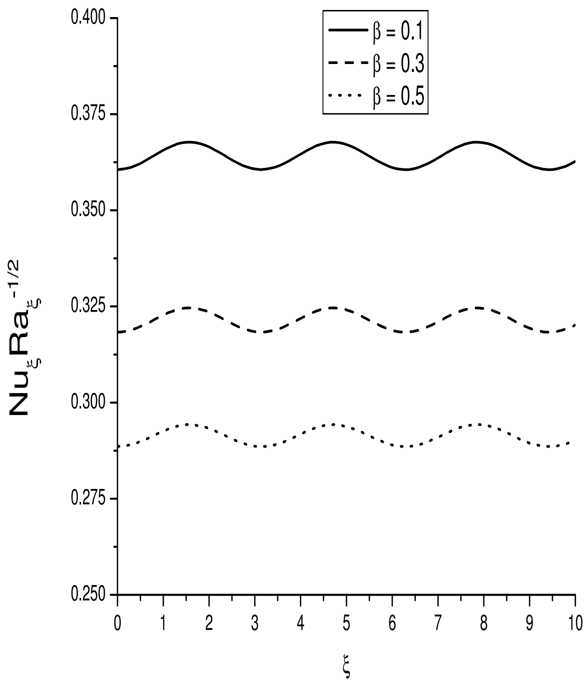

Figure 16: Local Nusselt number verses ξ for various values of β for $Sr = 1, N = 1, Le = 1, Du = 0.1, a = 0.5$ and $\theta = 3$

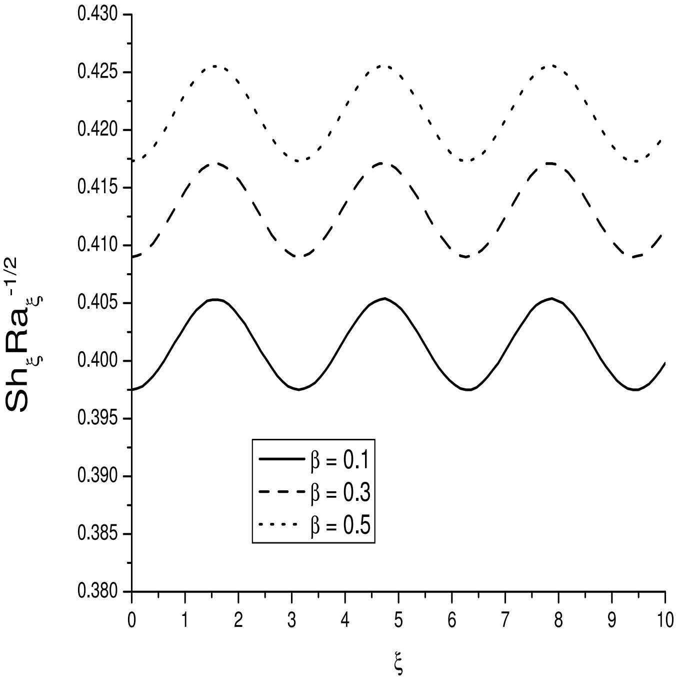

Figure 17: Local Sherwood number verses ξ for various values of β for $Sr = 1, N = 1, Le = 1, Du = 0.1, a = 0.5$ and $\theta = 3$

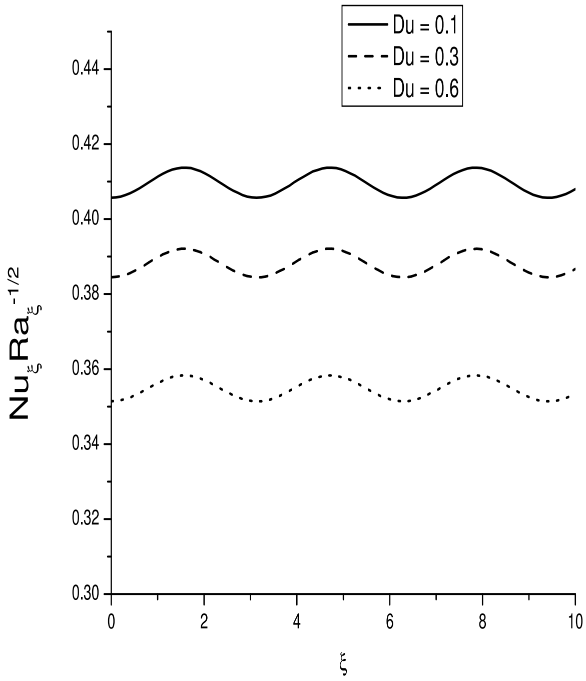

Figure 18: Local Nusselt number verses ξ for various values of Du for $Sr = 1, N = 1, Le = 1, \beta = 0.5, a = 0.5$ and $\theta = 3$

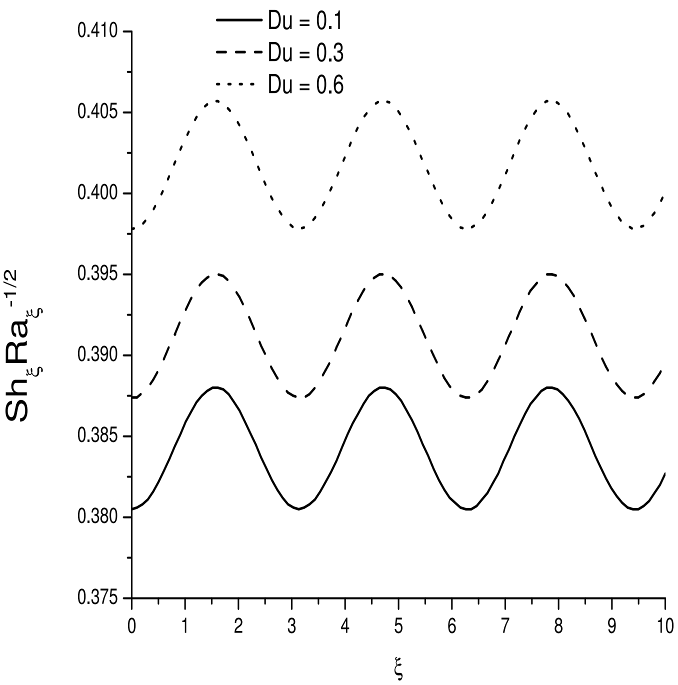

Figure 19: Local Sherwood number verses ξ for various values of Du for $Sr = 1, N = 1, Le = 1, \beta = 0.5, a = 0.5$ and $\theta = 3$

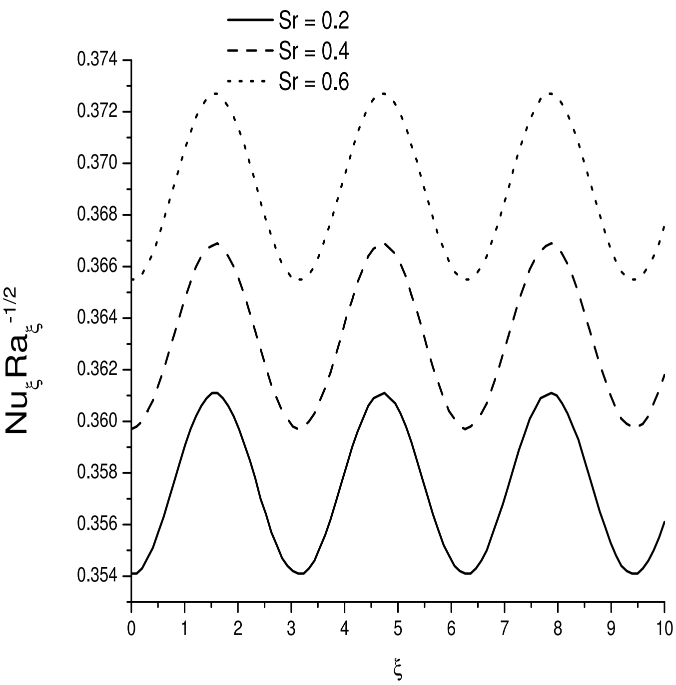

Figure 20: Local Nusselt number verses ξ for various values of Sr for $Du = 0.1, N = 1, Le = 1, \beta = 0.5, a = 0.5$ and $\theta = 3$

Table 1: Comparison of Local Nusselt number and the local Sherwood number calculated by the present method and that of Cheng [15] for $a = 0$, $\beta = 0$, $Du = 0, Sr = 0$ and $\theta_r \to \infty$

		$Nu_x Ra_x^{-1/2}$		$Sh_x Ra_x^{-1/2}$	
Le	N	Cheng [15]	Present	Cheng [15]	Present
1	4	0.992	0.9923	0.992	0.9923
10	4	0.681	0.6809	3.290	3.2883
100	4	0.521	0.5208	10.521	10.5205
4	1	0.559	0.5558	1.358	1.3565
4	2	0.650	0.6510	1.624	1.6238
4	3	0.728	0.7275	1.852	1.8532

The effect of Dufour parameter (Du) on the velocity, temperature and concentration profiles with respect to η is shown in Figs.(8)-(10). From Fig. (8), it is note worthy that increasing Dufour parameter is to increase the velocity profile throughout the boundary layer. In Fig.(9), it is seen that the temperature profile increases with increasing values of Dufour parameter, leading to an increase in thermal boundary layer thickness. In Fig.(10), an increase in Dufour parameter causes a slight decrease in solutal boundary layer thickness, which turns to reduce the concentration profile. It is an important to note that temperature is highest at leading edge of the plate and asymptotically decrease to zero far away from the plate with boundary condition.

The effect of Soret parameter (Sr) on the velocity, temperature and concentration profiles with respect to η is shown in Figs.(11)-(13). From these figures we observe that as increase in Soret parameter due to the contribution of the temperature gradients to species diffusion, results an enhancement in velocity, temperature and concentration profiles. It is noticed from these figures that velocity, temperature and concentration of fluid particle value of 1 at the plate surface and then decrease slowly till it attains the minimum value of zero far away from the plate surface with increasing value of Soret parameter.

Fig.(14) illustrates the stream wise profile of the local Nusselt number for various values of θ_r. This shows that increasing the θ_r reduces the fluctuation of local Nusselt number with the stream wise coordinate ξ. Fig.(15) shows the effect of θ_r on local Sherwood number. It is clear that increasing the θ_r leads to a smaller fluctuations of the local Sherwood number with stream wise coordinate ξ. For a value of ξ, Nusselt number and Sherwood number decreases with increase in θ_r.

Fig.(16) plots the stream wise distribution of the local Nusselt number for different values of variable thermal conductivity. Results show that increase in the variable thermal conductivity leads to smaller fluctuations of the local Nusselt number with the stream wise coordinate ξ. Fig.(17) shows the stream wise distribution of the local Sherwood number for various values of variabel thermal conductivity parameter. It is observed that greater fluctuations of the local Sherwood number with increasing values of variable thermal conductivity parameter with stream wise coordinate ξ. In addition, for a value of ξ, Nusselt number decreases and Sherwood number increases with increasing values of β.

Fig.(18) shows the stream wise distribution of the local rate of heat transfer for different values of Dufour parameter. As the Dufour parameter increases the fluctuations of the local rate of heat transfer with stream wise coordinate ξ is reduced. In addition, we observed that increase in Dufour parameter tends to increase the rate of heat transfer as increase in ξ. Fig.(19) depicts the stream wise distribution of the local sherwood number for various values of Dufour effect. It is observed that increase in the Dufour effect leads to a greater fluctuations of the local Sherwood number. More over, the Sherwood number increases with increasing values of Dufour parameter as ξ value increase.

The effect Soret parameter on Nusselt number is exhibits in Fig.(20). From this figure it is observed that increasing the Soret parameter results greater fluctuation in local Nusselt number with stream wise coordinate ξ. i.e., the rate of heat transfer increases with increasing values of Soret parameter. The effect of Soret parameter on local Sherwood number with stream wise distribution is shown in Fig.(21), it reveals that an increase in the Soret parameter leads to a smaller fluctuations of local Sherwood number with stream wise coordinate ξ. It means that the Sherwood number decreases with increase in Soret parameter.

Figs.(22)-(23) deals with the variation of the amplitude of the wavy surface a on Nusselt number and Sherwood number. It is clear that the rate of heat transfer and mass transfer decrease for the decrease of the amplitude of the wavy surface with increase in ξ. Nevertheless increasing the amplitude of the wavy surface results an enhancement in Nusselt number and Sherwood number. For $a = 0$ the model treated as vertical flat plate.

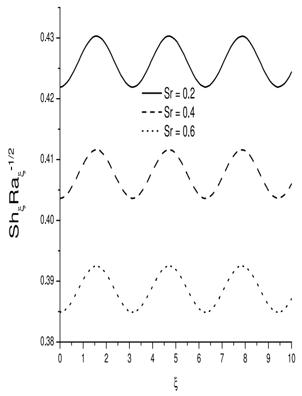

Figure 21: Local Sherwood number verses ξ for various values of Sr for $Du = 0.1, N = 1, Le = 1, \beta = 0.5, a = 0.5$ and $\theta = 3$

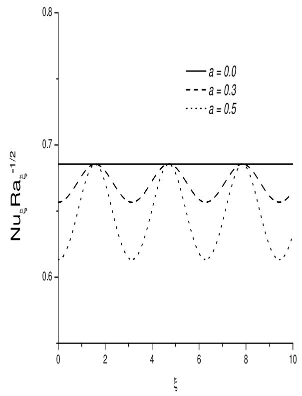

Figure 22: Local Nusselt number verses ξ for various values of wavy ratio a for $Du = 0.1, N = 1, Le = 1, \beta = 0.5, Sr = 1$ and $\theta = 3$

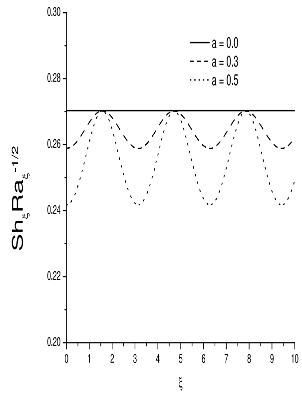

Figure 23: Local Sherwood number verses ξ for various values of wavy ratio a for $Du = 0.1, N = 1, Le = 1, \beta = 0.5, Sr = 1$ and $\theta = 3$

4 Conclusions

We have investigated the influence of variable properties and cross diffusion on steady convective heat and mass transfer flow over a vertical wavy surface embedded in a fluid saturated porous medium. The Fourth order Runge-Kutta method with Shooting technique is employed to solve the boundary layer equations and the numerical results are presented to analyze to fluid flow velocity, heat and mass transfer characteristics, Nusselt number and Sherwood number for various physical parameters. The conclusions of the present study is as follows:

- An increase in variable viscosity θ_r, decelerate the flow velocity, temperature, concentration and Nusselt number but enhance the Sherwood number.

- An increase in thermal conductivity parameter β enhance the temperature and Sherwood number but decrease in velocity, concentration and Nusselt number.

- As increase in Dufour parameter resulting in higher velocity, temperature and Sherwood number but decrease in concentration and Nusselt number. Velocity, temperature and concentration increase due to increase in Soret parameter.

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