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Discount Rate with Unstable Economic and Environmental Changes

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Abstract: The spread of the COVID-19 in 2020 has caused great losses to the global economic development. Meanwhile, global warming is still continuing, the economy and environment are facing instability. In order to obtain a more reasonable discount rate paradigm, this paper takes economic and environmental changes into account comprehensively, and gets a discount rate model under the condition of economic and environmental instability. It is analyzed that the discount rate value with a long period will be relatively small when facing the fluctuation of the total economic growth rate. The research results of this paper provide a theoretical support for the current policy of environmental protection and carbon emission limitation.

Keywords: Economic uncertainty; Climate change; Environmental uncertainty; Discount rate

1 Introduction

The Blue Paper on China's Climate Change (2020) shows[1] that the temperature in 2019 is about 1.1°C higher than the global pre-industrial level. Many key indicators of China's climate system show an accelerating trend of change, the global warming trend continues. When facing the challenge, the consensus of scientists from all over the world is that the decisions and choices made by mankind today will definitely affect the direction of climate change. As one of the most controversial issues in the economics of climate change, the selection of discount rate is undoubtedly important in order to determine the climate policy or climate action of contemporary humans towards future generations.

Meanwhile, after COVID-19 swept the world, global economic development was greatly impacted by the epidemic. In the first half of 2020, global GDP decreased by 4.3% compared with the same period last year[2]. However, various countries and regions promote their economies through monetary policies, so the OECD's outlook on the global economy is still optimistic[3] and predicts that the global economy is likely to recover to a normal level by the end of 2021. As a result, the global economy, like climate change, is highly volatile.

Based on the current complex situation of global changes, whether the discount rate can reflect the instability of global economic and climate change is a necessary consideration for the reasonable discount rate in the studying process.The discount rate originated from Frank Ramsey's A Mathematical Theory of Saving, which standardizing the normal form of the rate[4]. And research on climate change has become the focus since the 21st century. Stern Review[5] has caused controversy over the value of discount rate, and many economists such as Nordhaus[6] and Weitzman[7] have published papers, giving various opinions on the value of discount rate. Since then, studies on the discount rate have been numerous: Andersen [8]considers the elimination of risk and time preference, and discusses the robustness of hyperbolic discounting based on the method of exponential discounting; Johnson's[9] research is that in artificially delayed discounting, the discount rate will decrease as the reward range increases; Scandizzo[10] models the consumption and production under the influence of climate change as geometric Brownian motion, and also obtains a decreasing discount rate. According to the study of discount rate, it is found that nearly a quarter of the countries in the world use hyperbolic discount, and the setting of discount rate in each country is closely related to economy and culture[11]. The policy advice from economists is to use a discount rate that decreases over time[12].

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This paper creatively considers the fluctuation of global economy and climate in terms of growth rate, and incorporates the unstable situations such as large economic fluctuation and continuous climate warming into the discount rate model, and further determines the paradigm of global discount rate, which provides sufficient theoretical basis for the formulation of economic and climate policies.

2 Total economic growth rate with unstable economy and environment

2.1 Assumptions of economic and environmental instability

For the global economy, growth is normally steady. However, in the course of long-term economic development, there are various risks. For example, the COVID-19 has brought about a sharp slowdown in economic growth, and countries have adopted policies and market regulation to revive the economy. All these indicate that risks will lead to instability in economic development. Therefore, changes in medium - to long-term economic growth rates are unstable, and so, obviously, are changes in medium - to long-term climatic and environmental growth rates.

In order to enhance their economic strength, countries are focusing their attention on science and technology, resources and ecological environment. Among them, the link between ecological environment and economic strength is the most striking, because economic development and ecological protection seem to be in opposition at present. Human survival and the continuation of future generations require a good ecological environment. To achieve this goal, the global economy must make a choice: should it pursue national or regional economic status without considering the environmental losses caused by economic development temporarily, or limit economic development to maximize the protection of ecological environment?

In order to avoid one-sided pursuit of the economy or the environment, this article comprehensively considers economic and environmental changes[13], the global economic growth is defined as a linear combination of the net growth rate of economic and environmental growth

$$g_t = k_1 * g_t^{PureEconomy} + k_2 * g_t^{Enviroment} \tag{1}$$

Where $g_t^{PureEconomy}$ and $g_t^{Enviroment}$ respectively represent pure economic growth rate and environmental growth rate, and $k_1, k_2 \in [0, 1]$. The total amount of global market consumption is divided into economic consumption amount and environmental quality investment amount, so the total economic growth rate is a linear combination of pure economic growth rate and environmental growth rate.

The environmental growth rate here should be understood as the growth rate of the amount of environmental investment in the national economy after the government makes relevant climate policies.It is assumed that: The greater the investment in ecological environment, the better the environmental quality will be; The whole world complies with the Paris agreement[14] or some established climate protection policies, all countries and regions should invest in the ecological environment. Therefore, the rate of environmental growth in this paper is positive ($g_t^{Enviroment} > 0$).

While the global implementation of climate protection policies, the investment returns of environmental quality are lagging behind, and the current generation's investment cannot obtain real-time returns. Therefore, if the investment quota of environmental quality is set too large and the economic investment is ignored, the serious consequences such as slow economic development or even retrogression will not be affordable for contemporary people. Therefore, the economic growth rate in this paper is positive ($g_t^{PureEconomy} > 0$). Due to the particularity of signs, this paper sets corresponding initial values for pure economic and environmental growth rate: μ_x and μ_y, which are understood as averages of pure economic and environmental growth rates from the current period back up to a few years.

In addition, since the economic investment quota and environmental quality investment quota occupy different shares in the total market consumption, the corresponding growth rate also occupies different proportions of α and $1 - \alpha (\alpha \in [0, 1])$, so the total growth rate is

$$g_t = (\alpha * g_{xt} + \mu_x) + [(1 - \alpha) * g_{yt} + \mu_y]. \tag{2}$$

Among them, the growth rate of the current period will be adjusted based on the growth rate of the previous period, which is carried out by iteration, so

$$\begin{aligned} g_{xt} &= \beta * g_{x,t-1} + \epsilon_{g_{xt}}, \\ g_{yt} &= (1 - \beta) * g_{y,t-1} + \epsilon_{g_{yt}}. \end{aligned} \tag{3}$$

When adjusting for pure economic and environmental growth rates, policymakers in the current period need to take into account the specific circumstances of the global economy and climate change at the time, which are unpredictable.

Therefore, the unpredictable changes are random fluctuations, which we define as the disturbance term of the growth rate and subject to normal distribution, $\epsilon_{g_{xt}} \sim N(0, \sigma_x^2)$, $\epsilon_{g_{yt}} \sim N(0, \sigma_y^2)$.

In addition, different proportions of the pure economic and environmental growth rate in the last period are also assigned to β and $1-\beta(\beta \in [0,1])$ to reflect the fact that both of them jointly constitute the total growth rate but have differences. Therefore, it is more reasonable to assume that the disturbance terms of economic growth rate and environmental growth rate are independent. When $\beta = 0$ or $\beta = 1$, the pure economic or environmental growth rate in the current period will become a random variable and follow a normal distribution. At this time, the model is completely biased to the environmental growth rate or the pure economic growth rate, and the amount of investment in the environment or the pure economic will increase.

2.2 Total economic growth rate

When the total economic growth rate and the total amount of market consumption meet the relationship of exponential growth[15], $c_{t+1} = c_t * e^{g_t}$. So at time t, the total economic growth rate is

$$\begin{aligned}
\bar{g_t} &= lnc_t - lnc_0 \\
&= g_0 + g_1 + ... + g_{t-1} \\
&= \alpha\beta g_{x,-1}\frac{1-\beta^{t-1}}{1-\beta} + \alpha\epsilon_{g_{xt}}\sum_{r=1}^{t}\frac{1-\beta^{t-1}}{1-\beta} + t\mu_x \\
&+ (\alpha)(\beta)g_{y,-1}\frac{1-(1-\beta)^{t-1}}{\beta} + (1-\alpha)\epsilon_{g_{yt}}\sum_{r=1}^{t}\frac{1-(1-\beta)^{t-1}}{\beta} + t\mu_y.
\end{aligned} \tag{4}$$

As $\epsilon_{g_{xt}}$ and $\epsilon_{g_{yt}}$ are both normally distributed, the total economic growth rate at time t $\bar{g_t}$ is also normally distributed, and its mean is

$$\begin{aligned}
&E(lnc_t - lnc_0) \\
&= \alpha\beta g_{x,-1} * \frac{1-\beta^{t-1}}{1-\beta} + t\mu_x \\
&+ \alpha\beta g_{y,-1} * \frac{1-(1-\beta)^{t-1}}{\beta} + +t\mu_y.
\end{aligned} \tag{5}$$

The variance is

$$\begin{aligned}
&\frac{1}{t}Var(lnc_t) \\
&= \frac{\alpha^2\mu_x^2}{(1-\beta)^2} * [1 - 2\beta\frac{\beta^t - 1}{t(\beta-1)} + \beta^2\frac{\beta^{2t}-1}{t(\beta^2-1)}] \\
&+ \frac{(1-\alpha)^2\mu_y^2}{\beta^2} * [1 + 2(1-\beta)\frac{(1-\beta)^t - 1}{t\beta} + (1-\beta)^2\frac{(1-\beta)^{2t}-1}{t\beta(\beta-2)}].
\end{aligned} \tag{6}$$

When $t \to \infty$, the variance of total economic growth rate is

$$\frac{\alpha^2\mu_x^2}{(1-\beta)^2} + \frac{(1-\alpha)^2\mu_y^2}{\beta^2}. \tag{7}$$

When t is finite time, the variance is

$$\alpha^2\mu_x^2 + (1-\alpha)^2\mu_y^2. \tag{8}$$

By comparing Equation (7) with Equation (8), it can be easily found that the former is greater than the latter. This means that over time, the impact of fluctuations brought by the disturbance term of growth rate on the total economic growth rate is superimposed, and the longer the time is, the higher the instability of growth rate will be.

3 Discount model

Starting from the no-arbitrage principle[16], the formula for determining discount rate can be obtained, and the discount rate $r(t)$ is

$$r(t) = \delta - \frac{1}{t} ln \frac{U'(c_t)}{U'(c_0)}. \tag{9}$$

The utility function is taken as a common power utility function, and the discount rate is rewritten as

$$r(t) = \delta - \frac{1}{t} lnE[e^{-\eta(lnc_t - lnc_0)}]. \tag{10}$$

By using Arrow-Pratte Lemma[17], we can get

$$r(t) = \delta + \eta t^{-1} E[e^{-\eta(lnc_t - lnc_0)}] - 0.5\eta^2 t^{-1} Var(lnc_t) \tag{11}$$

By substituting into Equation (5) and Equation (6), it can be simplified as follows

$$\begin{aligned} r(t) = {} & \delta + \eta(\mu_x + \mu_y) - 0.5\eta^2 [\frac{\alpha^2 \mu_x^2}{(1-\beta)^2} + \frac{(1-\alpha)^2 \mu_y^2}{\beta^2}] \\ & + \eta[\alpha\beta g_{x,-1} \frac{1-\beta^{t-1}}{t(1-\beta)} + \alpha\beta g_{y,-1} \frac{1-(1-\beta)^{t-1}}{t\beta}] - 0.5\eta^2 \{\frac{\alpha^2 \mu_x^2}{(1-\beta)^2} [\beta^2 \frac{\beta^{2t}-1}{t(\beta^2-1)} - 2\beta \frac{\beta^t - 1}{t(\beta-1)}]\} \\ & - 0.5\eta^2 \{\frac{(1-\alpha)^2 \mu_y^2}{\beta^2} [2(1-\beta) \frac{(1-\beta)^t - 1}{t\beta} + (1-\beta)^2 \frac{(1-\beta)^{2t} - 1}{t\beta(\beta-2)}]\}. \end{aligned} \tag{12}$$

It can be seen that,the last three items on the right side of Equation (12) are all decreasing functions of t, when $t \to \infty$, there can be

$$r_\infty = \delta + \eta(\mu_x + \mu_y) - 0.5\eta^2 [\frac{\alpha^2 \mu_x^2}{(1-\beta)^2} + \frac{(1-\alpha)^2 \mu_y^2}{\beta^2}]. \tag{13}$$

On the right side of Equation (13), the first term is the pure time preference rate, which represents the degree of impatience of investment decision makers. The greater the value is, the more the decision makers tend to invest. The second is the wealth utility under the condition of diminishing marginal utility: premising that the future market is bound to be better than the current one, whether contemporary policy makers should consume as much as possible to ensure the long-term maximization of the total utility of human beings. The third is that investment policymakers choose low consumption in order to cope with the high instability of growth rate caused by the long interstices. In general, the discount rate over a long period will be small.

4 Conclusions

This paper divides the total amount of global market consumption into economic consumption and environmental quality investment. In economics, consumption and investment seem to be opposite concepts. But when applied to the economics of climate change, economic consumption is a good term, and environmental investment is not a strange one. Consumption in the environment is exactly an investment in environmental quality, and there is no contradiction here. Similarly, in the economics of climate change, economy and environment seem to be in opposition. This paper weakens other areas of market consumption and highlights economic consumption and environmental investment, which are the two major aspects that need to be paid attention to. It will be the philosophical question of "development or survival".

Through the research, we obtained the discount rate model under the uncertain conditions of economy and environment, analysing that the rate would be relatively small when faced with the fluctuation, which provided another theoretical support for the current implementation of environmental protection and carbon emission policies. However, the research on discount rate in this paper is still very preliminary and only stays in the stage of discount rate model. It is still necessary to carry out real value calibration and application to get a more perfect model.

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