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Modeling and Analysis of the Coupling Network of Interactions between Organisms

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Abstract: This paper studies a three-group model of biological interaction based on a coupling network, that is the one-predatorand two-prey model. First, the individual species are taken as network nodes, and the interaction between species is taken as edges. The mathematical model of the interaction between individuals with biological significance on the coupling network is established. Secondly, useing complex network theory to study the stability of the mathematical model. Using mathematical analysis methods, the basic intake rate of predator survival R_0 is calculated. If $R_0 > 1$, the positive equilibrium is locally and globally asymptotically stable. By constructing suitable Lyapunov function and the LaSalle's invariant principal, the global stability of positive equilibrium of the model is investigated in some conditions.

Keywords: coupled network; predator-prey model; Lyapunov function; LaSalle's invariant principal

1 Introduction

Population ecology is the focus of biological research, and useing mathematical theories and ideas to build models has become an important means of biological research[1][2]. In order to study the interaction between organisms, some scholars have used various population models to analyze population changes. In 1789, Malthus proposed an exponential growth model, which believed that the number of populations is not a simple additive relationship, but an exponential growth[3][4]. In 1838, Forhurst proposed a logistics model to describe populations. Gauss also proved through experiments that exponential growth is an excessively ideal situation. Many organisms will maintain a stable population after exponential growth for a period of time [5][6]. Literature[7][8][9]considers the effects of time, environment, and human activities on the population based on ordinary differential equations, and literature[10][11] considers the divergence, singularity, fractal and invasion process in biological system models and observations. In population ecology, mathematical models describing population dynamics play an important role in protecting the sustainable development of resources [12][13]. In traditional population dynamics, it is usually assumed that individuals are evenly distributed and the resources allocated by all individuals are equal. This too idealistic assumption makes the analysis tractable but unrealistic. In many practical situations, the difference between individuals in biological populations is a frequently encountered problem, which poses a great challenge to the traditional theoretical analysis that only deals with static targets. The complex network provides a good tool for depicting the complex individual connections in the population, so the population model can be considered as a complex network. A complex network is a large-scale network with complex topology and dynamic behavior, consisting of many interconnected nodes. The structural characteristics of the ecosystem are similar to the coupling network. The coupling network is a complex and uncertain system with self-organization, self-adaptation, quasi-balance, cooperative evolution and non-linear changes and other characteristics. With the continuous application of complex network theory, many real-life propagation phenomena can be studied using propagation models based on complex networks. The relationship between predation and prey in the ecosystem can be represented by the directed network between the two subnets in the coupling network, and the rest of the interspecies relationships such as competition and reciprocity can be represented by the undirected network on each subnet in the coupling network[14][15][16]. So the ecosystem and the coupling network have good compatibility. Use the coupling network method to establish the interaction model between biological populations, and use the propagation dynamics method of the complex network to study

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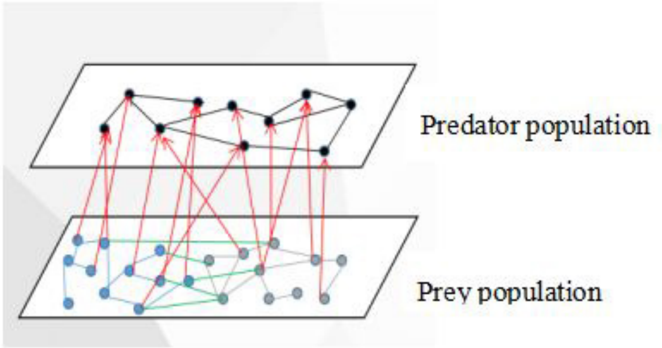

Figure 1: Unidirectional coupling network

the dynamic evolution of individual populations. This article mainly studies the biological model of the inter-population relationship on the coupling network.

2 Model description

The coupling network studied in this paper is composed of two subnets X and Y, each subnet is composed of nodes and edges connecting nodes. The subnet X represents the predator population, the subnet Y represents the prey population, the nodes represent the individuals of the population, and the edges connected between nodes represent the interactions between the individuals of the population, such as competition and reciprocity between individuals. In addition, in the subnet there are cross-connections between X and Y, such as predation between individuals. In other words, nodes on each subnet have two connection methods. The population individuals on one subnet can prey on the population individuals on the other subnet, but the opposite is not true. We call the above mentioned coupling network is unidirectional coupling network [17].

The unidirectional coupling network can be used to represent the energy flow between trophic levels, and the subnet X and subnet Y respectively represent two trophic levels respectively. In the food chain, energy can only flow from low nutritional level to high nutritional level, so there is no energy transfer between individuals on each subnet. In the ecosystem, there are competition, predation, reciprocity and other relationships among various populations, and there are density constraints within each population. Therefore, in a unidirectional coupling network. The relationship between nodes on each subnet represents the individual population, because individuals of higher trophic level can prey on individuals of lower trophic level, so the relationship between the subnets represents the predation effect of the higher trophic level on the lower trophic level. In this paper consider a unidirectional coupled network model of a predator and two prey. As shown in Figure 1.

In Figure 1, the degree (i, j, k) indicates that the nodes in the network have i edges connected to the population X, j edges connected to the prey population Y_1 in the subnet Y, and k edges connected to the prey population Y_2 in the subnet Y. That is, any individual in the area, the number of predator population X is i, the number of possible prey population Y_1 is j, and the number of predator population Y_2 that may be contacted is k. $(i, j, \cdot)$ means that the node has i edges connected to the predator in the subnet X, j edges is connected to the prey population Y_1 in the subnet Y, and any edges is connected to the prey population Y_2 in the subnet Y.

Consider three populations in a certain area. For simplicity, the competition between individuals and populations is not considered, and the number of edges from subnet X to Y is the same as the number of edges from subnet Y to X in the entire network.

Using the idea of the warehouse model, the three populations are divided into two warehouses, that is, two subnets. The population X is the upper trophic level, and the population Y_1 and the population Y_2 are the next trophic level. The population model is used to analyze the energy transmission on the unidirectional coupling network. In this model, because there is a predator relationship in the population X, Y_1, Y_2, the individual predators in the subnet X are connected to the prey individuals in the subnet Y. If there is an edge connection between the predator and the prey on the subnet, then the probability that the prey Y_1 is eaten by the predator is λ_1, and the probability that the prey Y_2 is eaten by the predator is λ_2. And suppose the natural growth rate of the population Y_1 is r_1, the natural growth rate of the population

Y_2 is r_2, and the natural mortality rate of the population X is d.

Based on this, we can get the following dynamic model of the semidirected network:

$$\begin{cases} \dfrac{dX(i,j,k)}{dt} = \lambda_1 j Y_1(i,j,k)\Theta_1 + \lambda_2 k Y_2(i,j,k)\Theta_2 - dX(i,j,k) \\ \dfrac{dY_1(i,j,k)}{dt} = -\lambda_1 j Y_1(i,j,k)\Theta_1 + r_1 Y_1(i,j,k) \\ \dfrac{dY_2(i,j,k)}{dt} = -\lambda_2 k Y_2(i,j,k)\Theta_2 + r_2 Y_2(i,j,k) \end{cases} \tag{1}$$

Where $\Theta_1 = \sum\limits_{i,j,k} \frac{jp(i,j,k)}{\langle j \rangle} X(i,j,k)$ indicates the probability that a prey with degree (i,j,k) in individual Y_1 touches the predator, $\Theta_2 = \sum\limits_{i,j,k} \frac{kp(i,j,k)}{\langle k \rangle} X(i,j,k)$indicates the probability that a prey with degree (i,j,k) in individual Y_2 touches the predator.

3 Equilibria and existence condition

The relative density of predators is $x_{(i,j,k)} = \frac{X_{(i,j,k)}}{N_{(i,j,k)}}$, the relative density of the prey Y_1 is $y_{1(i,j,k)} = \frac{Y_{1(i,j,k)}}{N_{(i,j,k)}}$, the relative density of the prey Y_2 is $y_{2(i,j,k)} = \frac{Y_{2(i,j,k)}}{N_{(i,j,k)}}$. So can get

$$Y_1(i,j,k) \geqslant 0, Y_2(i,j,k) \geqslant 0$$

then

$$\Theta(t) > 0$$

and

$$X(i,j,k) = 1 - Y_1(i,j,k) - Y_2(i,j,k), i,j,k = 1,2,3,\cdots$$

Available from system (1)

$$\frac{d\left(X_{(i,j,k)}(t) + Y_{1(i,j,k)}(t) + Y_{2(i,j,k)}(t)\right)}{dt} = 0$$

and

$$x_{(i,j,k)}(t) + y_{1(i,j,k)}(t) + y_{2(i,j,k)}(t) = 1$$

So that the system becomes

$$\begin{cases} \dfrac{dY_{1(i,j,k)}(t)}{dt} = r_1 Y_{1(i,j,k)} - \lambda_1 j Y_{1(i,j,k)}\Theta_1 \\ \dfrac{dY_{2(i,j,k)}(t)}{dt} = r_2 Y_{2(i,j,k)} - \lambda_2 k Y_{1(i,j,k)}\Theta_2 \end{cases} \tag{2}$$

So

$$\begin{cases} Y_{1(i,j,k)} = \dfrac{\lambda_1 j \Theta_1 r_2}{r_1 r_2 + \lambda_1 j \Theta_1 r_2 + \lambda_2 k \Theta_2 r_1} \\ Y_{2(i,j,k)} = \dfrac{\lambda_2 k \Theta_2 r_1}{r_1 r_2 + \lambda_1 j \Theta_1 r_2 + \lambda_2 k \Theta_2 r_1} \end{cases}$$

where

$$\begin{cases} \Theta_1 = \dfrac{\sum\limits_{i,j,k} jp_{(i,j,k)}}{\langle j \rangle} \cdot X_{(i,j,k)} \\ \Theta_2 = \dfrac{\sum\limits_{i,j,k} kp_{(i,j,k)}}{\langle k \rangle} \cdot X_{(i,j,k)} \end{cases}$$

Then can get the equation

$$\begin{cases} \Theta_1 f_1(\Theta_1) = \Theta_1 \\ \Theta_2 f_2(\Theta_2) = \Theta_2 \end{cases}$$

where

$$\begin{cases} f_1(\Theta_1) = \dfrac{\sum\limits_{i,j,k} jp(i,j,k)}{\langle j \rangle} \cdot \dfrac{r_1 r_2}{r_1 r_2 + \lambda_1 j \Theta_1^* r_2 + \lambda_2 k \Theta_2^* r_1} \\ f_2(\Theta_2) = \dfrac{\sum\limits_{i,j,k} kp(i,j,k)}{\langle k \rangle} \cdot \dfrac{r_1 r_2}{r_1 r_2 + \lambda_1 j \Theta_1^* r_2 + \lambda_2 k \Theta_2^* r_1} \end{cases}$$

Because

$$f'(\Theta) < 0, f(1) < 1$$

when

$$\begin{cases} \Theta_1 f_1(\Theta_1) = \Theta_1 \\ \Theta_2 f_2(\Theta_2) = \Theta_2 \end{cases}$$

if and only if

$$\begin{cases} f_1(0) > 1 \\ f_2(0) > 1 \end{cases}$$

obtain

$$\begin{cases} \dfrac{\sum\limits_{i,j,k} jp(i,j,k)}{\langle j \rangle} \cdot \dfrac{r_1 r_2}{r_1 r_2 + \lambda_2 k \Theta_2^* r_1} = R_1 > 1 \\ \dfrac{\sum\limits_{i,j,k} kp(i,j,k)}{\langle k \rangle} \cdot \dfrac{r_1 r_2}{r_1 r_2 + \lambda_1 j \Theta_1^* r_2} = R_2 > 1 \end{cases}$$

Hence

$$R_0 = \max\{R_1, R_2\}$$

therefore when $R_0 > 1$, system (2) has the only positive balance point

$$E^*\left(Y_{1(1,1,1)}^*, Y_{2(1,1,1)}^*, \cdots, Y_{1(i,j,k)}^*, Y_{2(i,j,k)}^*\right)$$

So the positive equilibrium solution of system (1) is

$$\begin{cases} X^*_{(i,j,k)} = \dfrac{r_1 r_2}{r_1 r_2 + \lambda_1 j \Theta_1^* r_2 + \lambda_2 k \Theta_2^* r_1} \\ Y_{1(i,j,k)}^* = \dfrac{\lambda_1 j \Theta_1^* r_2}{r_1 r_2 + \lambda_1 j \Theta_1^* r_2 + \lambda_2 k \Theta_2^* r_1} \\ Y_{2(i,j,k)}^* = \dfrac{\lambda_2 k \Theta_2^* r_1}{r_1 r_2 + \lambda_1 j \Theta_1^* r_2 + \lambda_2 k \Theta_2^* r_1} \end{cases}$$

4 Dynamical Analysis of the Model

Theorem 1 *When all the eigenvalues of the Jacobian matrix of system (1) have negative real parts, the positive equilibrium point of system (1) is locally asymptotically stable.*

According to the system equation can obtain the ordinary equilibrium point $(0, 0, 0)$ and the positive equilibrium point (X^*, Y_1^*, Y_2^*).

Because the ordinary equilibrium point indicates that there is no predator or prey in this area, it has no research significance, so it is discarded.

Next, prove the local stability of the positive equilibrium point (X^*, Y_1^*, Y_2^*). List the Jacobian matrix of the system (1) at the positive equilibrium point as:

$$J = \begin{bmatrix} F_{(1,1,1)}^{(1,1,1)} & \cdots & F_{(1,1,1)}^{(1,1,k)} & \cdots & F_{(1,1,1)}^{(1,j,k)} & \cdots & F_{(1,1,1)}^{(i,j,k)} \\ \vdots & & \vdots & & \vdots & & \vdots \\ F_{(1,1,k)}^{(1,1,1)} & \cdots & F_{(1,1,k)}^{(1,1,k)} & \cdots & F_{(1,1,k)}^{(1,j,k)} & \cdots & F_{(1,1,k)}^{(i,j,k)} \\ \vdots & & \vdots & & \vdots & & \vdots \\ F_{(1,j,k)}^{(1,1,1)} & \cdots & F_{(1,j,k)}^{(1,1,k)} & \cdots & F_{(1,j,k)}^{(1,j,k)} & \cdots & F_{(1,j,k)}^{(i,j,k)} \\ \vdots & & \vdots & & \vdots & & \vdots \\ F_{(i,j,k)}^{(1,1,1)} & \cdots & F_{(i,j,k)}^{(1,1,k)} & \cdots & F_{(i,j,k)}^{(1,j,k)} & \cdots & F_{(i,j,k)}^{(i,j,k)} \end{bmatrix}$$

where $F_{(l,m,n)}^{(i,j,k)}$ represents the third-order matrix obtained by deriving the differential equation $X_{(l,m,n)}, Y_{1(l,m,n)}, Y_{2(l,m,n)}$ with degree (l,m,n) to $X_{(i,j,k)}, Y_{1(i,j,k)}, Y_{2(i,j,k)}$ with degree (i,j,k).

When the degree satisfies $(l,m,n) = (i,j,k)$, there is

$$F_{(i,j,k)}^{(i,j,k)} = \begin{bmatrix} \lambda_1 j Y_{1(i,j,k)} \frac{\sum_{i,j,k} jp_{(i,j,k)}}{\langle j \rangle} + \lambda_2 k Y_{2(i,j,k)} \frac{\sum_{i,j,k} kp_{(i,j,k)}}{\langle k \rangle} - d & \lambda_1 j \Theta_1 & \lambda_1 k \Theta_2 \\ -\lambda_1 j Y_{1(i,j,k)} \frac{\sum_{i,j,k} jp_{(i,j,k)}}{\langle j \rangle} & r_1 - \lambda_1 j \Theta_1 & 0 \\ -\lambda_2 k Y_{2(i,j,k)} \frac{\sum_{i,j,k} kp_{(i,j,k)}}{\langle k \rangle} & 0 & r_2 - \lambda_1 k \Theta_2 \end{bmatrix}$$

When the degree satisfies $(l,m,n) \neq (i,j,k)$, there is

$$F_{(l,m,n)}^{(i,j,k)} = \begin{bmatrix} \lambda_1 j Y_{1(i,j,k)} \frac{\sum_{i,j,k} jp_{(i,j,k)}}{\langle j \rangle} + \lambda_2 k Y_{2(i,j,k)} \frac{\sum_{i,j,k} kp_{(i,j,k)}}{\langle k \rangle} & 0 & 0 \\ -\lambda_1 j Y_{1(i,j,k)} \frac{\sum_{i,j,k} jp_{(i,j,k)}}{\langle j \rangle} & 0 & 0 \\ -\lambda_2 k Y_{2(i,j,k)} \frac{\sum_{i,j,k} kp_{(i,j,k)}}{\langle k \rangle} & 0 & 0 \end{bmatrix}$$

After calculation, we can get $F_{(l,m,n)}^{(i,j,k)} = 0$, so the Jacobian matrix of system (1) is a diagonal matrix.

The characteristic value $x_{(i,j,k)} = F_{(i,j,k)}^{(i,j,k)}$ is obtained from $|xE - J| = 0$, so whether the characteristic polynomial has a negative real part eigenvalue depends on whether the value of $F_{(i,j,k)}^{(i,j,k)}$ is negative. After calculation, the similarity matrix of the matrix J is J^*, so $|xE - J| = |xE - J^*| = 0$, the elements on the diagonal of J^* are:

$$F^{*(i,j,k)}_{(i,j,k)} = \begin{bmatrix} -d & r_1 & r_2 \\ -\lambda_1 j Y^*_{1(i,j,k)} \frac{\sum_{i,j,k} jp_{(i,j,k)}}{\langle j \rangle} & r_1 - \lambda_1 j \Theta_1^* & 0 \\ -\lambda_2 k Y^*_{2(i,j,k)} \frac{\sum_{i,j,k} kp_{(i,j,k)}}{\langle k \rangle} & 0 & r_2 - \lambda_1 k \Theta_2^* \end{bmatrix}$$

Substituting the positive balance point into the matrix and calculating it can be obtained, when $r_1 < \lambda_1, r_2 < \lambda_2$, the matrix $F^{*(i,j,k)}_{(i,j,k)}$, so the eigenvalues of the characteristic polynomial $|xE - J| = 0$ have negative real parts, so the positive balance point is locally asymptotically stable [18].

Theorem 2 *If $R_0 > 1, r_1 < \lambda_1, r_2 < \lambda_2$, the positive equilibrium of system (1) is globally asymptotically provided.*

Proof Consider the following Lyapunov function

$$\begin{aligned} V(t) = \frac{1}{2} \sum_{i,j,k} \left\{ a_{(i,j,k)} \left(Y_{1(i,j,k)}(t) - Y^*_{1(i,j,k)}(t) \right)^2 \right\} + \frac{1}{2} \sum_{i,j,k} \left\{ b_{(i,j,k)} \left(Y_{2(i,j,k)}(t) - Y^*_{2(i,j,k)}(t) \right)^2 \right\} \\ + \Theta_1(t) - \Theta_1^* - \Theta_1^* \ln \frac{\Theta_1(t)}{\Theta_1^*} + \Theta_2(t) - \Theta_2^* - \Theta_2^* \ln \frac{\Theta_2(t)}{\Theta_2^*} \end{aligned} \tag{3}$$

the following solve the function for the differential form:

$$\begin{aligned} V'(t) &= \sum_{i,j,k} a_{(i,j,k)} \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big) Y_{1(i,j,k)}'(t) + \sum_{i,j,k} b_{(i,j,k)} \Big(Y_{2(i,j,k)}(t) - Y_{2(i,j,k)}^{*}(t) \Big) Y_{2(i,j,k)}'(t) \\ &+ \frac{\Theta_1 - \Theta_1^*}{\Theta_1} \Theta_1' + \frac{\Theta_2 - \Theta_2^*}{\Theta_2} \Theta_2' \\ &= V_1 + V_2 + V_3 + V_4 \end{aligned} \tag{4}$$

where

$$\begin{aligned} V_1 &= \sum_{i,j,k} a_{(i,j,k)} \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big) Y_{1(i,j,k)}' \\ &= \sum_{i,j,k} a_{(i,j,k)} \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big) \left(-\lambda_1 j Y_{1(i,j,k)}(t) \Theta_1 + r_1 Y_{1(i,j,k)}(t) \right) \end{aligned} \tag{5}$$

From

$$-\lambda_1 j Y_{1(i,j,k)}^{*}(t) \Theta_1^* + r_1 Y_{1(i,j,k)}^{*}(t) = 0$$

we can get

$$\begin{aligned} V_1 &= \sum_{i,j,k} a_{(i,j,k)} \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big) Y_{1(i,j,k)}' \\ &= \sum_{i,j,k} a_{(i,j,k)} \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big) \Big(-\lambda_1 j \Big(Y_{1(i,j,k)}(t) \Theta_1 - Y_{1(i,j,k)}^{*}(t) \Theta_1^* \Big) + r_1 \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big) \Big) \\ &= \sum_{i,j,k} a_{(i,j,k)} \Big(r_1 \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big)^2 \\ &+ \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big) \Big(-\lambda_1 j \Big(Y_{1(i,j,k)}(t) \Theta_1 - Y_{1(i,j,k)}^{*}(t) \Theta_1 + Y_{1(i,j,k)}^{*}(t) \Theta_1 - Y_{1(i,j,k)}^{*}(t) \Theta_1^* \Big) \Big) \Big) \\ &= \sum_{i,j,k} a_{(i,j,k)} \Big(r_1 \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big)^2 \\ &- \lambda_1 j \Theta_1 \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big)^2 - \lambda_1 j Y_{1(i,j,k)}^{*} \Big(Y_{1(i,j,k)}(t) - Y_{1(i,j,k)}^{*}(t) \Big) (\Theta_1 - \Theta_1^*) \Big) \end{aligned}$$

$$\begin{aligned} V_2 &= \sum_{i,j,k} b_{(i,j,k)} \Big(Y_{2(i,j,k)}(t) - Y_{2(i,j,k)}^{*}(t) \Big) Y_{2(i,j,k)}' \\ &= \sum_{i,j,k} b_{(i,j,k)} \Big(Y_{2(i,j,k)}(t) - Y_{2(i,j,k)}^{*}(t) \Big) \Big(-\lambda_2 k \Big(Y_{2(i,j,k)}(t) \Theta_2 - Y_{2(i,j,k)}^{*}(t) \Theta_2^* \Big) + r_2 \Big(Y_{2(i,j,k)}(t) - Y_{2(i,j,k)}^{*}(t) \Big) \Big) \\ &= \sum_{i,j,k} b_{(i,j,k)} \Big(r_2 \Big(Y_{2(i,j,k)}(t) - Y_{2(i,j,k)}^{*}(t) \Big)^2 \\ &+ \Big(Y_{2(i,j,k)}(t) - Y_{2(i,j,k)}^{*}(t) \Big) \Big(-\lambda_2 k \Big(Y_{2(i,j,k)}(t) \Theta_2 - Y_{2(i,j,k)}^{*}(t) \Theta_2 + Y_{2(i,j,k)}^{*}(t) \Theta_2 - Y_{2(i,j,k)}^{*}(t) \Theta_2^* \Big) \Big) \Big) \\ &= \sum_{i,j,k} b_{(i,j,k)} \Big(r_2 \Big(Y_{2(i,j,k)}(t) - Y_{2(i,j,k)}^{*}(t) \Big)^2 \\ &- \lambda_2 k \Theta_2 \Big(Y_{2(i,j,k)}(t) - Y_{2(i,j,k)}^{*}(t) \Big)^2 - \lambda_2 k Y_{2(i,j,k)}^{*} \Big(Y_{2(i,j,k)}(t) - Y_{2(i,j,k)}^{*}(t) \Big) (\Theta_2 - \Theta_2^*) \Big) \end{aligned}$$

$$
\begin{aligned}
V_3 &= \frac{\Theta_1-\Theta_1^*}{\Theta_1}\Theta_1' = \frac{(\Theta_1-\Theta_1^*)}{\Theta_1}\frac{\sum jP_{(i,j,k)}}{\langle j\rangle}\left[\lambda_1 jY_{1(i,j,k)}\Theta_1+\lambda_2 kY_{2(i,j,k)}\Theta_2-dX_{(i,j,k)}\right]\\
&= \frac{(\Theta_1-\Theta_1^*)}{\Theta_1}\left[\frac{\sum jP_{(i,j,k)}}{\langle j\rangle}\left(\lambda_1 jY_{1(i,j,k)}\Theta_1+\lambda_2 kY_{2(i,j,k)}\Theta_2\right)-d\Theta_1\right]\\
&= (\Theta_1-\Theta_1^*)\left[\frac{\sum jP_{(i,j,k)}}{\langle j\rangle}\left(\lambda_1 jY_{1(i,j,k)}+\lambda_2 kY_{2(i,j,k)}\frac{\Theta_2}{\Theta_1}\right)-d\right]\\
&= (\Theta_1-\Theta_1^*)\frac{\sum jP_{(i,j,k)}}{\langle j\rangle}\left(\lambda_1 j\left(Y_{1(i,j,k)}-{Y_1}^*{}_{(i,j,k)}\right)+\lambda_2 k\left(Y_{2(i,j,k)}\frac{\Theta_2}{\Theta_1}-Y^*{}_{2(i,j,k)}\frac{\Theta_2^*}{\Theta_1^*}\right)\right)\\
V_4 &= \frac{\Theta_2-\Theta_2^*}{\Theta_2}\Theta_2' \quad = \frac{(\Theta_2-\Theta_2^*)}{\Theta_2}\frac{\sum kP_{(i,j,k)}}{\langle k\rangle}\left[\lambda_1 jY_{1(i,j,k)}\Theta_1+\lambda_2 kY_{2(i,j,k)}\Theta_2-dX_{(i,j,k)}\right]\\
&= \frac{(\Theta_2-\Theta_2^*)}{\Theta_2}\left[\frac{\sum kP_{(i,j,k)}}{\langle k\rangle}\left(\lambda_1 jY_{1(i,j,k)}\Theta_1+\lambda_2 kY_{2(i,j,k)}\Theta_2\right)-d\Theta_2\right]\\
&= (\Theta_2-\Theta_2^*)\left[\frac{\sum kP_{(i,j,k)}}{\langle k\rangle}\left(\lambda_1 jY_{1(i,j,k)}\frac{\Theta_1}{\Theta_2}+\lambda_2 kY_{2(i,j,k)}\right)-d\right]\\
&= (\Theta_2-\Theta_2^*)\frac{\sum kP_{(i,j,k)}}{\langle k\rangle}\left(\lambda_1 j\left(Y_{1(i,j,k)}\frac{\Theta_1}{\Theta_2}-{Y_1}^*{}_{(i,j,k)}\frac{\Theta_1^*}{\Theta_2^*}\right)+\lambda_2 k\left(Y_{2(i,j,k)}-Y^*{}_{2(i,j,k)}\right)\right)\\
&= \frac{\sum kP_{(i,j,k)}}{\langle k\rangle}\left(\lambda_1 j\left(Y_{1(i,j,k)}\frac{\Theta_1}{\Theta_2}-{Y_1}^*{}_{(i,j,k)}\frac{\Theta_1^*}{\Theta_2^*}\right)(\Theta_2-\Theta_2^*)\right.\\
&\quad\left.+\lambda_2 k\left(Y_{2(i,j,k)}-Y^*{}_{2(i,j,k)}\right)(\Theta_2-\Theta_2^*)\right)
\end{aligned}
$$

Let $a_{(i,j,k)}=b_{(i,j,k)}=\frac{j}{\langle j\rangle}+\frac{k}{\langle k\rangle}$, so we get:

$$
\begin{aligned}
V'(t) &= \sum\left(\frac{j}{\langle j\rangle}+\frac{k}{\langle k\rangle}\right)r_1\Big(Y_{1(i,j,k)}(t)-Y_{1(i,j,k)}^*\Big)^2+\sum\left(\frac{j}{\langle j\rangle}+\frac{k}{\langle k\rangle}\right)r_2\Big(Y_{2(i,j,k)}(t)-Y_{2(i,j,k)}^*\Big)^2\\
&\quad-\sum\left(\frac{j}{\langle j\rangle}+\frac{k}{\langle k\rangle}\right)\lambda_1 j\Theta_1\Big(Y_{1(i,j,k)}(t)-Y_{1(i,j,k)}^*\Big)^2-\sum\left(\frac{j}{\langle j\rangle}+\frac{k}{\langle k\rangle}\right)\lambda_2 k\Theta_2\Big(Y_{2(i,j,k)}(t)-Y_{2(i,j,k)}^*\Big)^2\\
&\quad+A
\end{aligned}
\tag{6}
$$

where

$$
\begin{aligned}
A &= \frac{1}{\langle j\rangle}\sum jP_{(i,j,k)}\lambda_2 k\left(Y_2\frac{\Theta_2}{\Theta_1}-{Y_2}^*\frac{{\Theta_2}^*}{{\Theta_1}^*}\right)(\Theta_1-{\Theta_1}^*)+\frac{1}{\langle k\rangle}\sum kP_{(i,j,k)}\lambda_1 j\left(Y_1\frac{\Theta_1}{\Theta_2}-{Y_1}^*\frac{{\Theta_1}^*}{{\Theta_2}^*}\right)(\Theta_2-{\Theta_2}^*)\\
&\quad-\frac{1}{\langle k\rangle}\sum kP_{(i,j,k)}\lambda_1 j\left(Y_1-{Y_1}^*\right)(\Theta_1-{\Theta_1}^*)-\frac{1}{\langle j\rangle}\sum jP_{(i,j,k)}\lambda_2 k\left(Y_2-{Y_2}^*\right)(\Theta_2-{\Theta_2}^*)
\end{aligned}
\tag{7}
$$

We simplify formula, and we combine the first and fouth items to get the following results:

$$
\begin{aligned}
A_1 &= \frac{1}{\langle k\rangle}\sum kP_{(i,j,k)}\lambda_1 j\left(Y_1\frac{\Theta_1}{\Theta_2}-{Y_1}^*\frac{{\Theta_1}^*}{{\Theta_2}^*}\right)(\Theta_2-{\Theta_2}^*)-\frac{1}{\langle k\rangle}\sum kP_{(i,j,k)}\lambda_1 j\left(Y_1-{Y_1}^*\right)(\Theta_1-{\Theta_1}^*)\\
&= \frac{1}{\langle k\rangle}\sum kP_{(i,j,k)}\lambda_1 j\frac{1}{{\Theta_2}^*\Theta_2}\left(\Theta_1{\Theta_2}^*-{\Theta_1}^*\Theta_2\right)\left({Y_1}^*\Theta_2-Y_1{\Theta_2}^*\right)
\end{aligned}
\tag{8}
$$

According to the definition of the model,it is not difficult to find $\Theta_1{\Theta_2}^*-\Theta_1^*\Theta_2=0$, so $A_1=0$. $A=0$ is correct for the same reason. So

$$
\begin{aligned}
V'(t) &\leqslant \sum\left(\frac{j}{\langle j\rangle}+\frac{k}{\langle k\rangle}\right)(r_1-\lambda_1)\Big(Y_{1(i,j,k)}(t)-Y_{1(i,j,k)}^*\Big)^2\\
&\quad+\sum\left(\frac{j}{\langle j\rangle}+\frac{k}{\langle k\rangle}\right)(r_1-\lambda_1)\Big(Y_{1(i,j,k)}(t)-Y_{1(i,j,k)}^*\Big)^2
\end{aligned}
\tag{9}
$$

When $r_1<\lambda_1$, $r_2<\lambda_2$, $V'=0$ if and only if $X_{(i,j,k)}=X_{(i,j,k)}^*, Y_{1(i,j,k)}(t)=Y_{1(i,j,k)}^*$ and $Y_{2(i,j,k)}(t)=Y_{2(i,j,k)}^*$. According to the Lyapunov theorem[19] and the LaSalles invariant principal[20], we can conclude that endemic equilibrium of system (1) is globally asymptotically stable.

5 Conclusion

This paper studies the predator-prey system between the three species under the coupled network. The conditions for the existence of the only positive equilibrium point of the predator-prey system are obtained by calculation, and it is also proved that when the basic intake rate is greater than 1, the positive equilibrium point of the predator-prey system is at a certain local and global asymptotical stability under conditions. That is to say, when the predators predation is greater than the energy consumed by its own growth, the predator population can reproduce steadily for a long time, which is more in line with our understanding of the growth of individual organisms and population numbers increasing conditions. Therefore, it is feasible to use the coupling network to study the interactions between individual populations, which provides a basis for studying the quantitative changes of biological populations, the protection and management of precious animals and plants. However, the competition between individual populations is not considered in this article, I hope it can be improved in the future.

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