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Sustainability and Substitutability of Simple Solvable Growth Models under Irreversible Climate Change

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Abstract: This paper establishes a simple and analytical discrete time growth model which includes expendable capital, human capital, non-renewable resources and irreversible greenhouse gas accumulation. The model allows the analysis of different elasticity of substitution between renewable and non-renewable factors of production. Under favorable substitution probabilities, consumption and wealth are either U-shaped or increase over time. Under less favorable substitution possibilities, for a small initial capital stock, consumption and wealth initially increase, peak after a finite time, and eventually decline at a constant rate. In the case of Cobb-Douglas production and logarithmic utility, consumption and wealth can take similarly characteristic paths, depending on the productivity of human capital accumulation. I further characterize the optimal carbon tax and show that the results extend to the case of random capital and resource dynamics.

Keywords:sustainability; substitutability; substitution and supplement; non-renewable resource use; irreversible climate change; random resource dynamics; optimal growth; climate policy

1 Introduction

More and more literature studies the economics of climate change and the use of exhaustible resources in quantitative comprehensive assessment models (Nordhaus 1993, Stern 2006 et al.) [1]. These models generally dont allow analytical solutions, which makes it difficult to evaluate the general properties of (numerically computed) model solutions and, in particular, almost impossible to perform global sensitivity analysis. This can be problematic because some important parameter values are largely unknown, including the time preference rate, the elasticity of intertemporal substitution, or the parameters of the climate damage function. Therefore, it is useful to build a model that is simple enough to provide closed solutions, but rich enough to cover the most relevant processes for studying issues such as climate change, the use of exhaustible resources, economic growth and sustainability. Among others, Golosov et al. (2014) [2], Gerlach and Lisky (2012) [3], breischgel and Kalidas (2013) [4] have developed this analytical model. To achieve a closed-form solution, these models usually assume that the production function is cobb-Douglas and the utility function is logarithmic. This may be too restrictive an assumption, as it is well known that the elasticity of substitution may play a crucial role in the likelihood of sustainable growth (Dasgupta and Heal, 1979) [5]. In this paper, I develop a simple, analytic discrete time growth model that allows analysis of different elasticity of substitution between renewable and non-renewable factors of production. The model's stock variables include two accumulated factors of production, an anthropogenic and consumptive capital stock and human capital, as well as two exhaustible resource/natural sinks, a non-renewable resource and accumulated greenhouse gases. Climate change is considered irreversible based on recent ipcc reports (Collins et al., 2013), as well as previous contributions by Stollery (1998) [6] and Bretschger and Karydas (2013). I present the complete analytical characteristics of the efficient and optimal dynamics in the model; In addition, I describe an effective and socially optimal carbon tax. I find that under favorable substitution probabilities, or in the case of Cobb-Douglas/logarithmic utility, for sufficiently productive human capital accumulation, consumption and wealth are either U-shaped or increase monotonously over time. Under the less favorable possibility of substitution, or, in the case of Cobb-Douglas/logarithmic

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utility, for human capital accumulation with low productivity, if the initial capital stock is not very large, consumption and wealth initially increase, peak after a finite time, and eventually decline at a constant rate.

2 Simple solvable growth models with irreversible climate change

In the following, I develop a discrete time model with four variables. There are two kinds of stock that can be accumulated, one is directly consumed K_t and the other is human capital H_t. In addition, there is a non-renewable resource. When this resource, interpreted as a fossil fuel such as coal or oil, is used to produce goods, the emissions it produces accumulate into a stock X_t that causes environmental damage as it contributes to climate change.

2.1 Theoretical Model

The main element of the model is the dynamics of capital stock .It is described by the following equation

$$\begin{aligned} K_{t+1} &= F\left(K_t - C_t, E_t, L_t, X_t\right) \\ &= \left[\alpha\left(K_t - C_t\right)^{1-\gamma} + \beta {E_t}^{1-\gamma} + \delta\left(\bar{X} - X_t\right)^{1-\gamma} + \left(1 - \alpha - \beta - \delta\right) {L_t}^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \end{aligned} \tag{1}$$

where$F\left(K_t - C_t, E_t, L_t, X_t\right)$ is a constant-returns-to-scale, constant-elasticity-of-substitution (CES) production function. For the parameter values I assume $\alpha, \beta, \gamma, \delta > 0, \alpha + \beta + \delta \leq 1$.Furthermore, C_t is consumption, E_t is resource input, and L_t is the input of labor force in human capital. The initial capital stock K_0 is given. For a while, consumption takes place before the production of goods, leaving only the surplus capital stock $K_t - C_t$ for production. The entire production output is then available for the capital stock in the next period $(t + 1)$, at which point part of it is again consumed and the remaining capital stock is used for production. Production output increased further in terms of input of resources E_t and human capital L_t .

The constant elasticity of substitution between any two inputs is given by $1/\gamma$.For $\gamma \geq 1$, all production inputs are necessary for production. In particular the resource is necessary for production, i.e. $F(K_t - C_t, E_t, L_t, X_t) = 0$. For $\gamma < 1$, the resource is not necessary for production. As long as $\gamma > 0$, however, consumption will never exceed the currently available capital stock $C_t < K_t$ and for $\delta > 0$, the stock of greenhouse gases will never exceed the upper limit, i.e. $X_t < \overline{X}$, where $\overline{X}$ is to be interpreted as a threshold value beyond which the stock of greenhouse gases causes intolerable damages. Note that for the special case , the production function (1) becomes the Cobb-Douglas function with exponents α for capital K_t , β for resource input E_t , δ for the stock of greenhouse gases X_t , $1 - \alpha - \beta - \delta$ for human capital L_t.

The stock of non-renewable resources follows the usual equation of motion

$$S_{t+1} = S_t - E_t \tag{2}$$

i.e. the amount of remaining resources S_t is simply reduced by the amount extracted. Because the total size of available resources is limited by the initial inventory S_0 , resource inputs in production must eventually be reduced. This tends to limit future consumption possibilities. Note that for $\alpha + \beta = 1$ and $\delta = 0$, the model established here is the discrete version of Dasgupta and Heal Solow-Stiglitz (DHSS) model (Dasgupta and Heal 1974, Solow 1974, Stiglitz 1974a, b) [7], which extends to consider human capital formation and climate disruption. For this case, constant utility can be expected to be valid for or for $\gamma \geq 1$ and $\alpha > \beta$. Constant utility is not feasible for $\gamma = 1$ considering irreversible climate damage ($\delta > 0$).

When human capital is productive , there is an ever-growing factor of production that can be produced independently of resources. Investment labour can be directed to any kind of capital stock. Now turn to the description of human capital dynamics. The total time endowment is one. Time can also be used to build up the stock of human capital

$$H_{t+1} = (1 - \varepsilon)H_t + \kappa(H_t - L_t) \tag{3}$$

Among them is the productivity of human capital accumulation, and is the debasement rate of human capital. Contrary to the input of non-renewable resources, the accumulation of human capital tends to increase the possibility of future consumption. We assume that in this way, if all the time is spent accumulating human capital and not producing goods, i.e. $L_t = 0$, then human capital can surely accumulate.

We adopt a very simple description of climate dynamics, using a state variable X_t, which is a serious simplification because it ignores the time lag between emissions and temperature rise. However, our focus is on the long-term sustainability impacts of climate change. In this regard, the most important point is that carbon emissions have an impact on long-term consumption possibilities. Specifically, we believe that climate damage is irreversible. This is consistent with previous studies, including Stollery(1998)[8] and Bretschger and Karydas(2013). More importantly, there is clear evidence from climate science that the consequences of carbon emissions are essentially irreversible (Collins et al. 2013). Under these assumptions, and with appropriate standardized units, the state of the climate system is described by the following dynamics

$$X_{t+1} = X_t - E_t \tag{4}$$

And the initial carbon budget. Therefore, intertemporal use of carbon is limited by two aspects: first, the stock of carbon resources initially available on land; The other is the initial budget for carbon emissions into the atmosphere. Along the optimal path, only the tighter of the two constraints is valid. Collins et al. (2013) and McGlade and Ekins(2015)[9] believe that the effective constraint is the remaining carbon budget. So let's assume. This means that the remaining carbon budget is a scarce resource, and an effective emission tax will depict the corresponding scarce rent (Asheim 2013) [10].
Consumption of goods produces utility, and the standard specification I use is that instantaneous utility is described by an isoelastic function

$$U\left(C_t\right) = \frac{{C_t}^{1-\theta}}{1-\theta} \tag{5}$$

With $\theta > 0$. In the limit $\theta \to 1$, the utility function becomes the logarithmic specification.
The socially optimal intertemporal distribution will be the distribution that maximizes the present value of utility discount (interpreted as some kind of discounted utilitarian welfare standard),

$$W = \sum_{t=0}^{\infty} \rho^t U\left(C_t\right) \tag{6}$$

Where $0 < \rho < 1$ is the utility discount factor. For the objective function (6), the parameter of the utility function (5) will be interpreted as the preference for intertemporal consumption smoothing, or the reciprocal of the intertemporal elasticity of substitution.

3 Market-balanced and effective climate policies

For describing the market equilibrium, I measure all prices in terms of the capital good K_t . Using to denote interest rate on capital; ω_t to denote the wage rate for human capital; q_t to denote to market price for resource input; and λ_t to denote an emission tax imposed by a regulator, the profit of a representative producing firm is

$$F(K_t, E_t, L_t) - (1+\gamma_t)K_t - (q_t + \lambda_t)E_t - \omega_t L_t \tag{7}$$

Since capital used in production must be returned to the owners of capital, the total "price" of capital inputs is $1 + \gamma_t$. The first order condition of profit maximization is the familiar condition that marginal product value equals the price of each factor.

$$F_{K_t} = 1 + \gamma_t \tag{8}$$

$$F_{E_t} = q_t + \lambda_t \tag{9}$$

$$F_{L_t} = \omega_t \tag{10}$$

Non-renewable resources are owned by a company that uses the capital interest rate as a discount rate to maximize the present value of profits:

$$\max_{E_t} \sum_{t=0}^{\infty} \frac{1}{(1+\gamma_t)^t} q_t E_t \tag{11}$$

The necessary and sufficient conditions for profit maximization are known as Hotelling's rule

$$q_t = (1+\gamma_t) q_{t-1} \tag{12}$$

Considering the equilibrium of the resource market, I can use (8) and (9) in (11). After a bit of rearrangement, this will lead to

$$-\frac{\lambda_t - F_{K_t}\lambda_{t-1}}{F_{E_{t-1}}} + \frac{F_{E_t} - F_{E_{t-1}}}{F_{E_{t-1}}} = F_{K_t} - 1 \tag{13}$$

The market equilibrium condition is compared with the efficiency condition $-\frac{F_{X_t}}{F_{E_{t-1}}} + \frac{F_{E_t} - F_{E_{t-1}}}{F_{E_{t-1}}} = F_{K_t} - 1$, and the following results are obtained Proposal 1. Any dynamic effective tax rate on greenhouse gas emissions has the following characteristics.

$$\lambda_{t-1} = -\frac{F_{X_t} - \lambda_t}{F_{K_t}} = -\sum_{\lambda=0}^{\infty} \frac{F_{X_{t+\lambda}}}{F_{K_{t+\lambda}}} \tag{14}$$

The explanation for the effective tax rate is obvious: it is equal to the present value of marginal damage caused by current greenhouse gas emissions to uncertain future production output.
The description of market equilibrium is accomplished by assuming that representative households are maximized under budget constraints (8)

$$K_{t+1} = (1 + \gamma_t)(K_t - C_t) + q_t E_t + \omega_t L_t + T_t \tag{15}$$

And (3) the accumulation of human capital as described. Here I also introduced a one-off tax transfer to consumers. The optimization of consumption paths leads to a discrete-time version of the familiar Keynesian-Ramsey rule

$$\frac{C_{t-1}^{-\gamma}}{\rho C_t^{-\gamma}} = 1 + \gamma_{t-1} \tag{16}$$

4 Conclusion

This paper establishes and analyzes a simple discrete time growth model, which includes man-made capital, expendable capital, human capital, non-renewable resources and irreversible climate change. I find that, with favourable substitution possibilities, the most likely outcome is permanent growth. Under less favorable alternative possibilities, or with relatively low productivity in human capital accumulation, the economy's most likely best path is an initial phase of growth, followed by a peak in consumption, followed by a long de-growth phase.

The advantage of the model considered in this paper is that it allows the analysis of changes in the elasticity of substitution between different renewable and exhaustible factors of production. The closed solution is possible only when the elasticity of substitution between different factors of production is equal to the elasticity of intertemporal substitution in consumption. Given the easiness of the analysis, the model allows for considerable extension. My focus in this paper is on optimal dynamics. A very interesting extension of the analysis could be the study of sub-optimal taxation or investment in human capital. One hypothesis in this regard might be that a higher than optimal level of emissions taxes, and a similar higher than optimal level of investment in human capital accumulation, can further delay peak consumption. In such cases, societies may face a trade-off between efficiency, the maximum present value of benefits, and equity, the idea that future generations will be relatively better off as peak consumption is delayed.

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